# PosterChild: Blend-Aware Artistic Posterization

Cheng-Kang Ted ChaoGeorge Mason UniversityKaran SinghUniversity of TorontoYotam GingoldGeorge Mason University











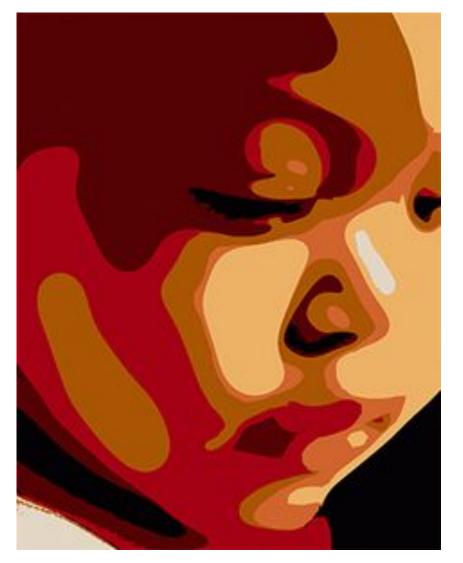








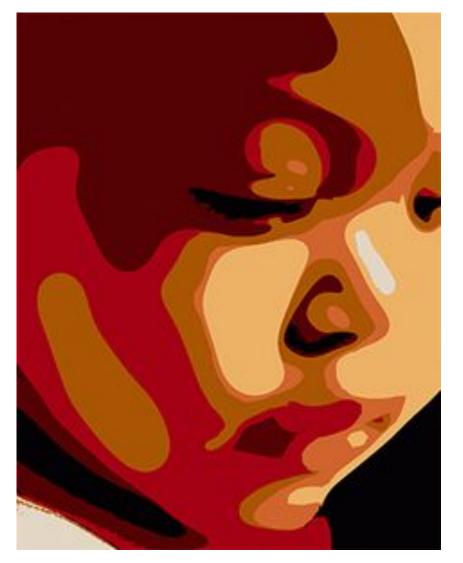




























Previous Work: [Xu and Kaplan 2008], [Gerstner et al. 2013], [Afifi 2018]

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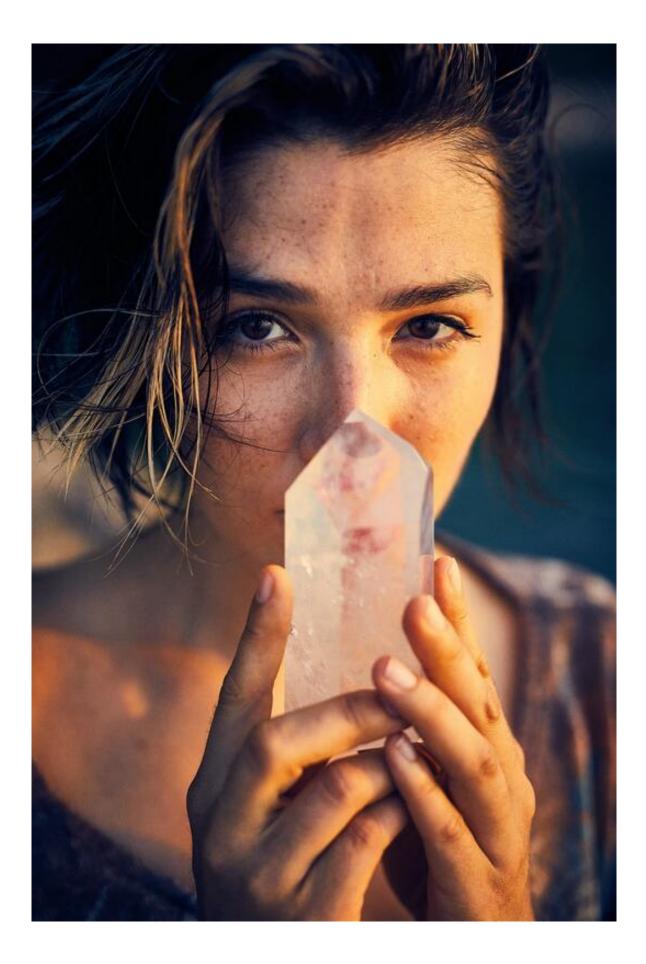
**Photoshop's Posterization Filter** 



**Artist's Creation** 

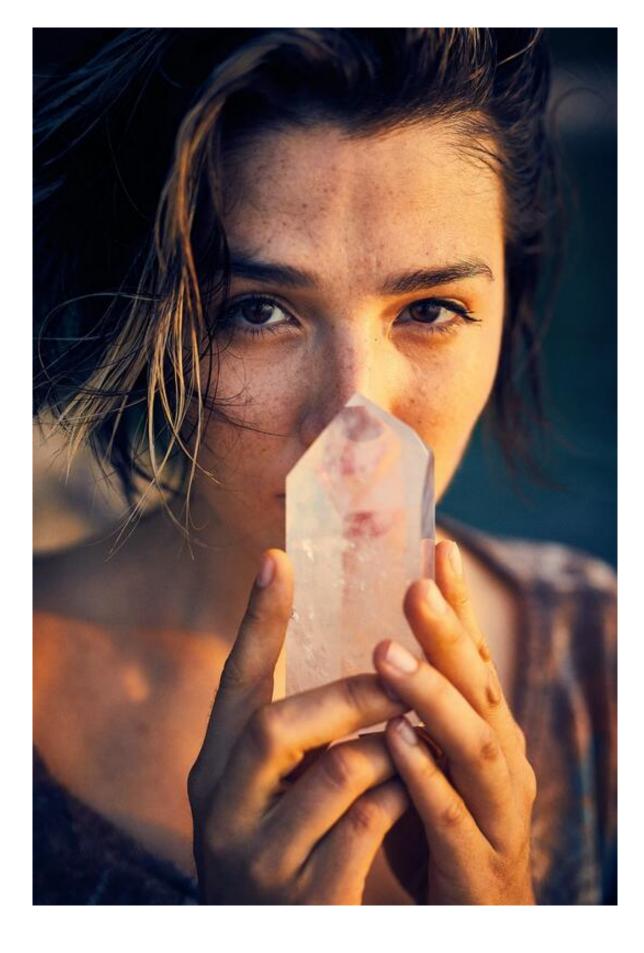
- Step 1: Choose a color palette
- Step 2: Form approximate solid-color regions
- Step 3: Improve region color blends
- Step 4: Improve region boundaries

- Step 1: Convex-hull based palette extraction
- Step 2: Form approximate solid-color regions
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- Step 4: Improve region boundaries



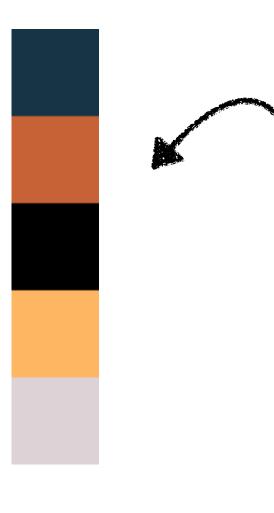


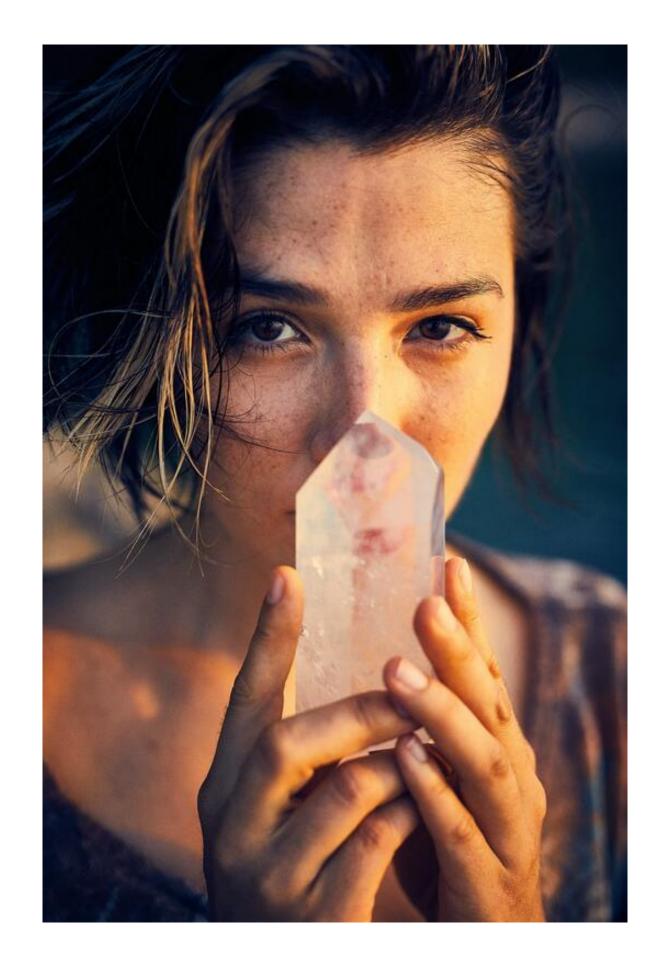
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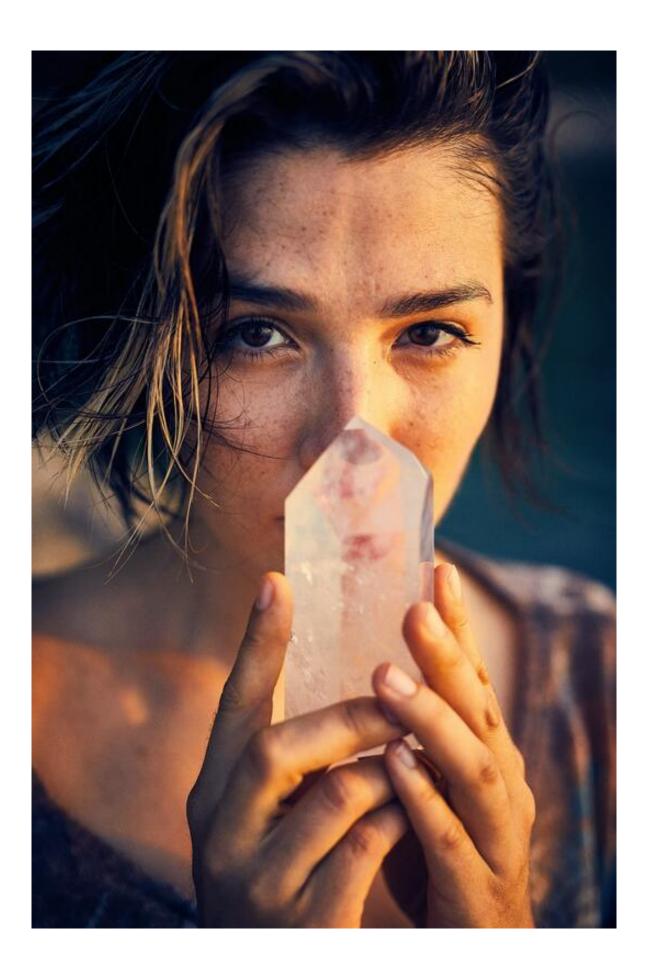






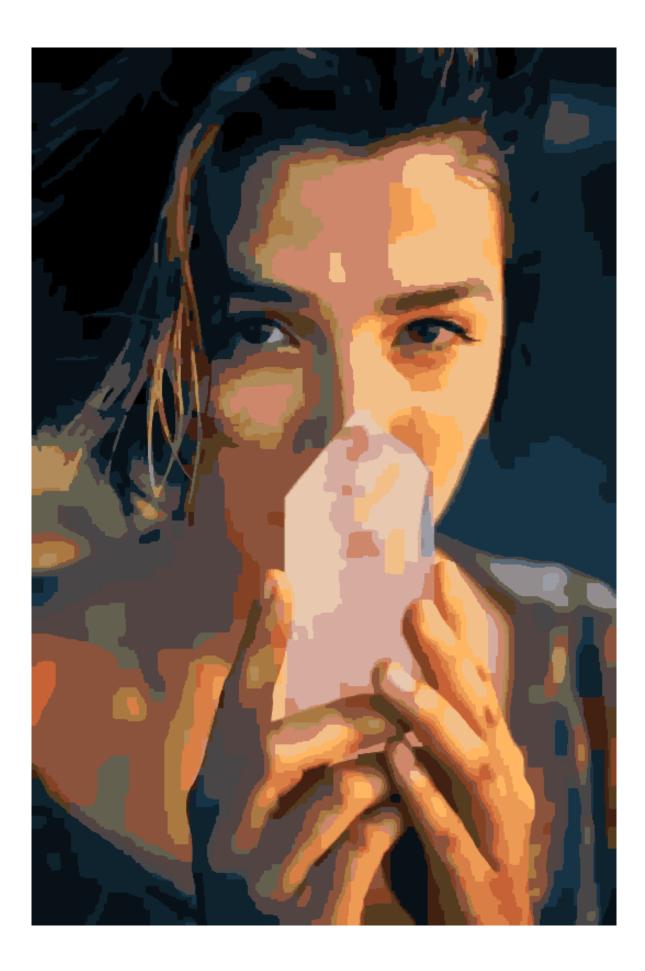
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- Step 1: Choose a color palette
- Step 2: Rough region and color assignment
- Step 3: Improve region color blends
- Step 4: Improve region boundaries





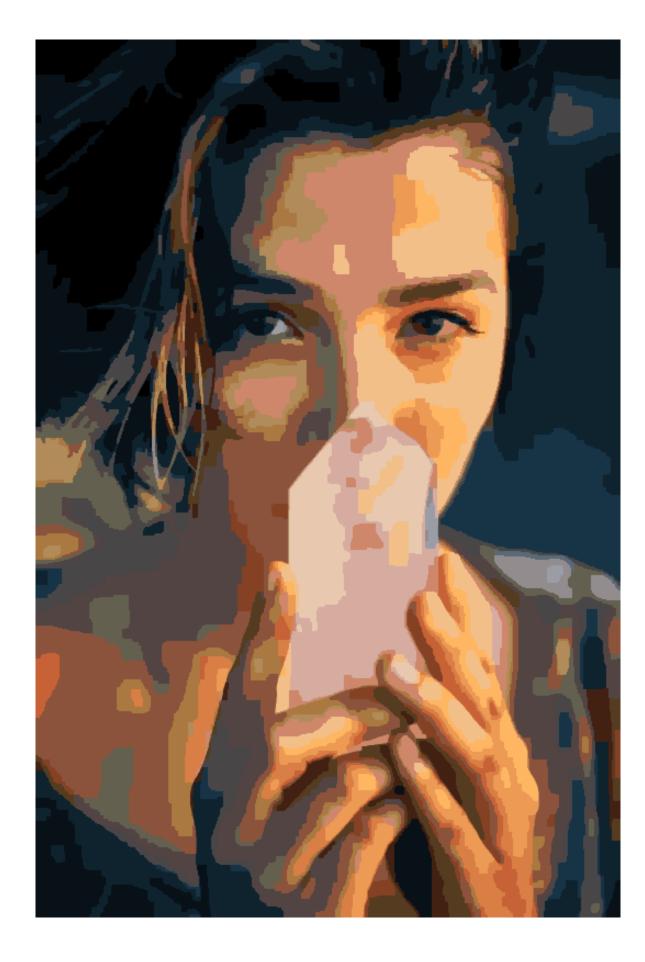
- Step 1: Choose a color palette
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- Step 3: Improve region color blends
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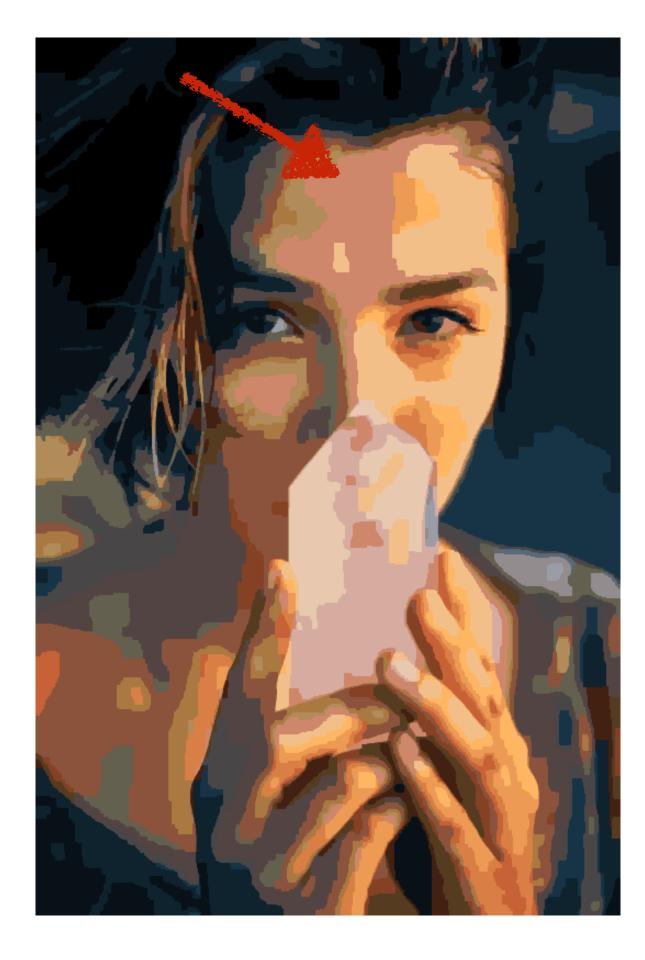


- Step 1: Choose a color palette
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- Step 3: Improve region color blends
- Step 4: Improve region boundaries

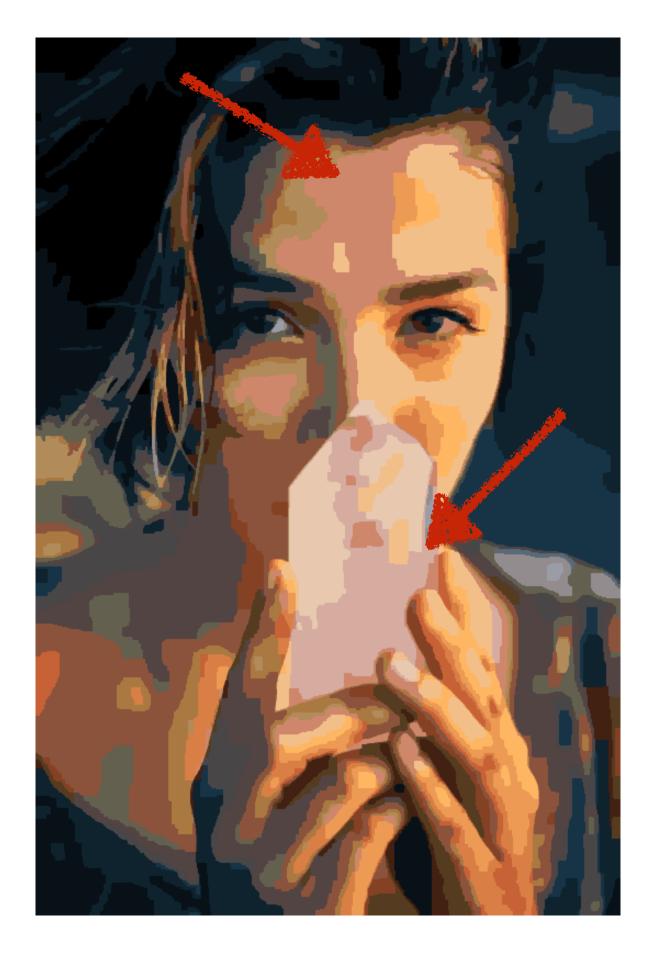
- Step 1: Choose a color palette
- Step 2: Form approximate solid-color regions
- Step 3: Blend refinement
- Step 4: Improve region boundaries



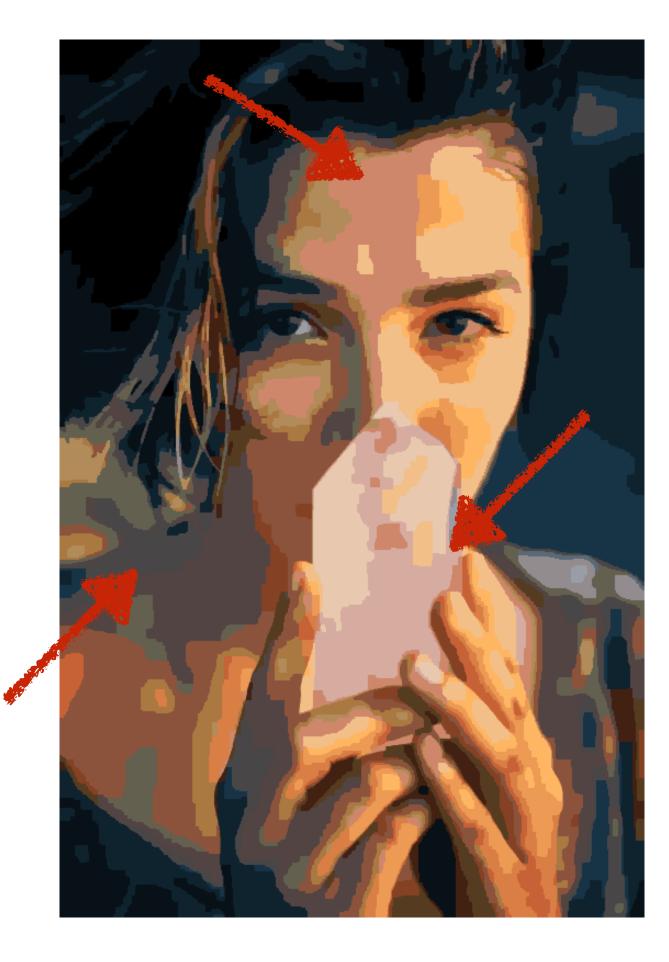
- Step 1: Choose a color palette
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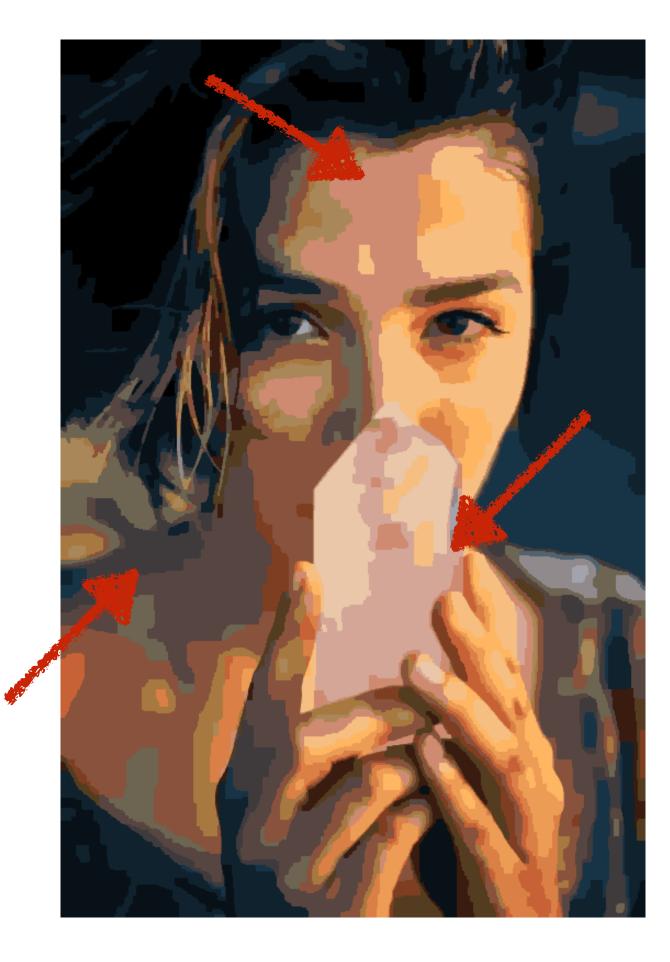
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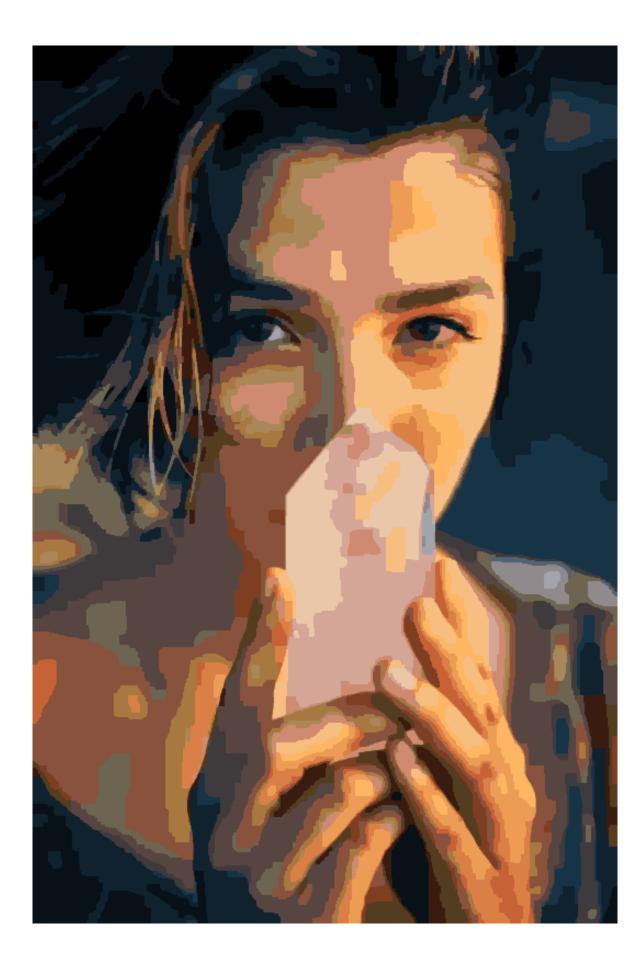


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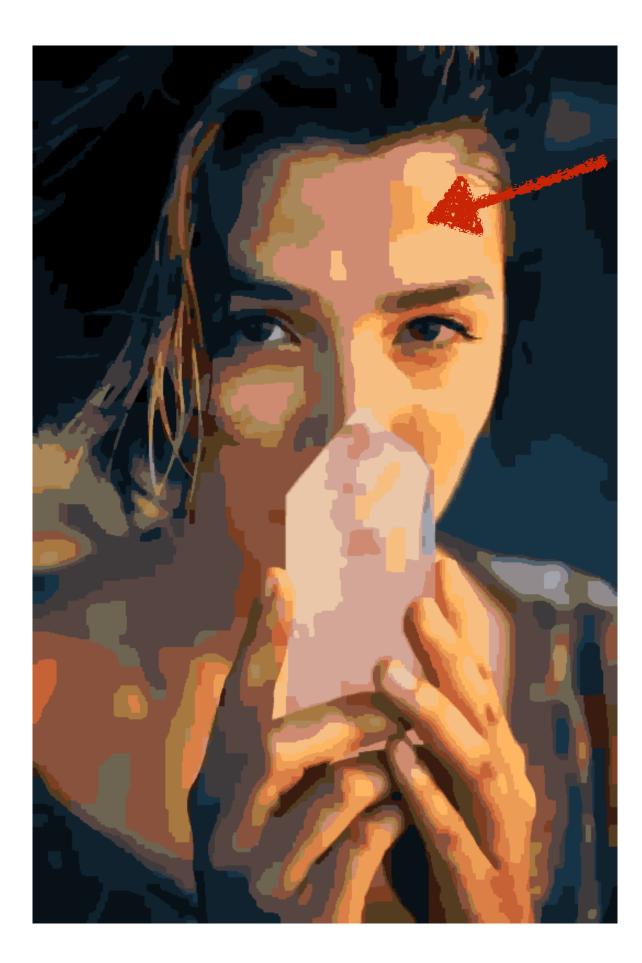


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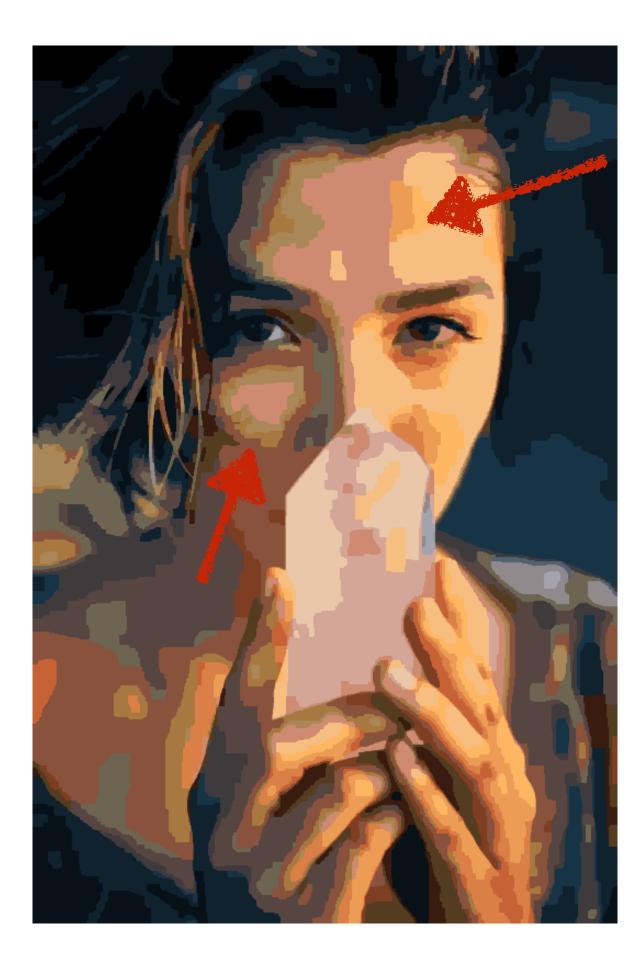
- Step 1: Choose a color palette
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- Step 4: Region boundary smoothing



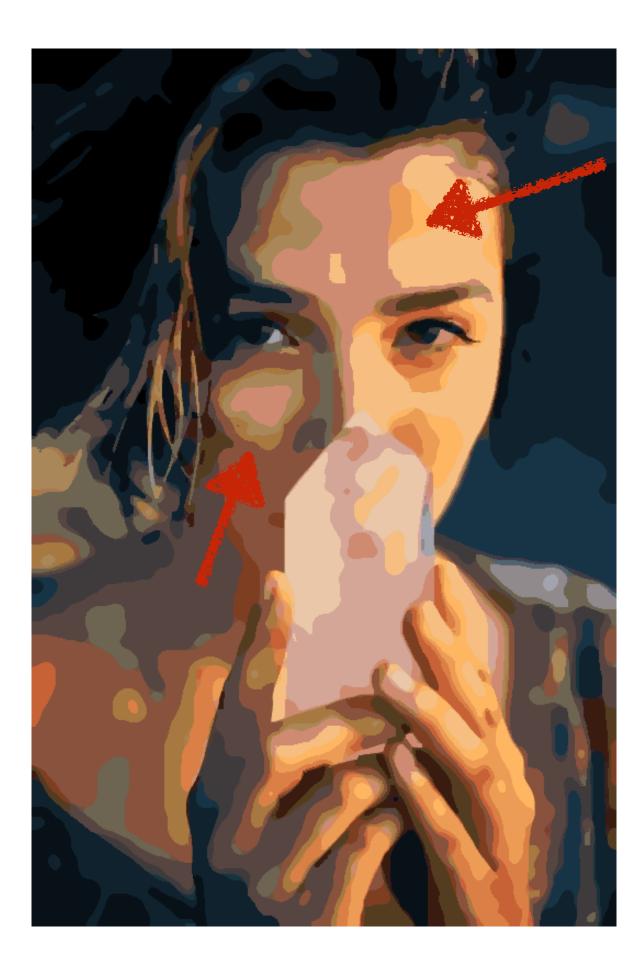
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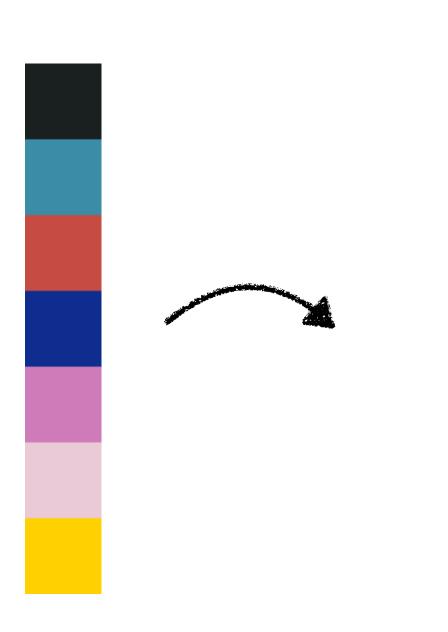
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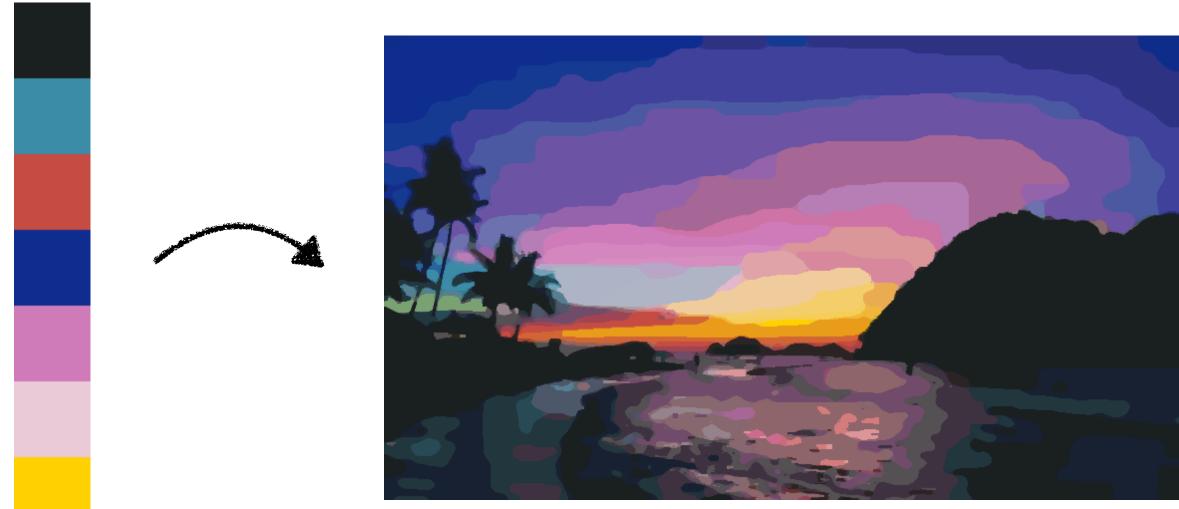
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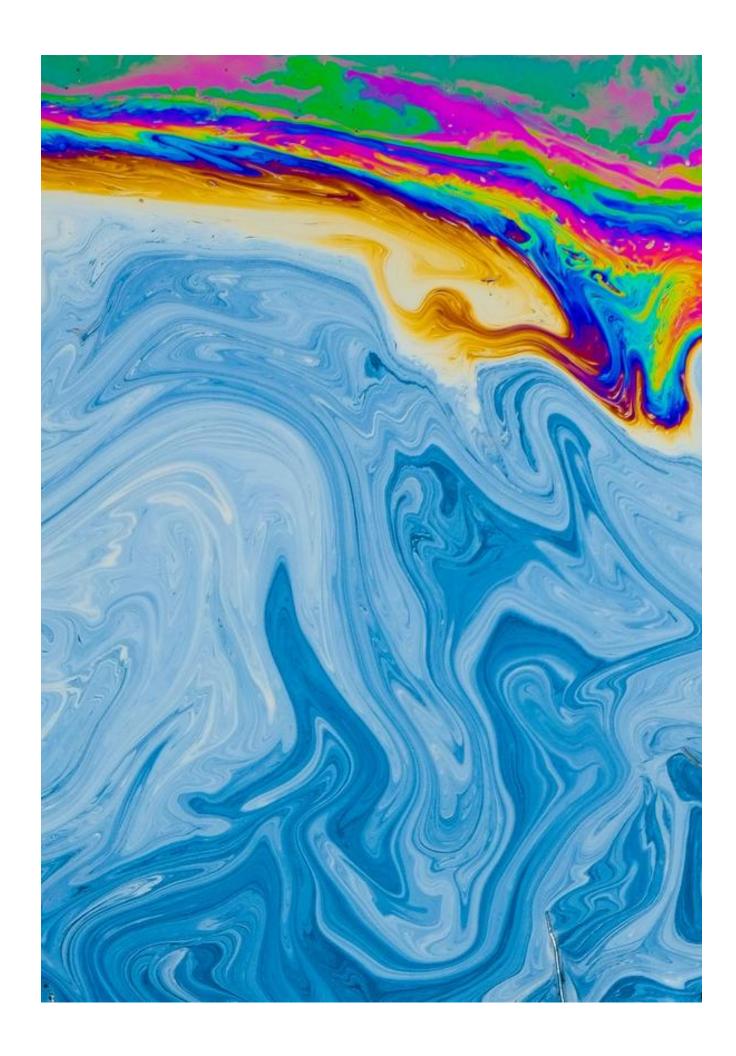
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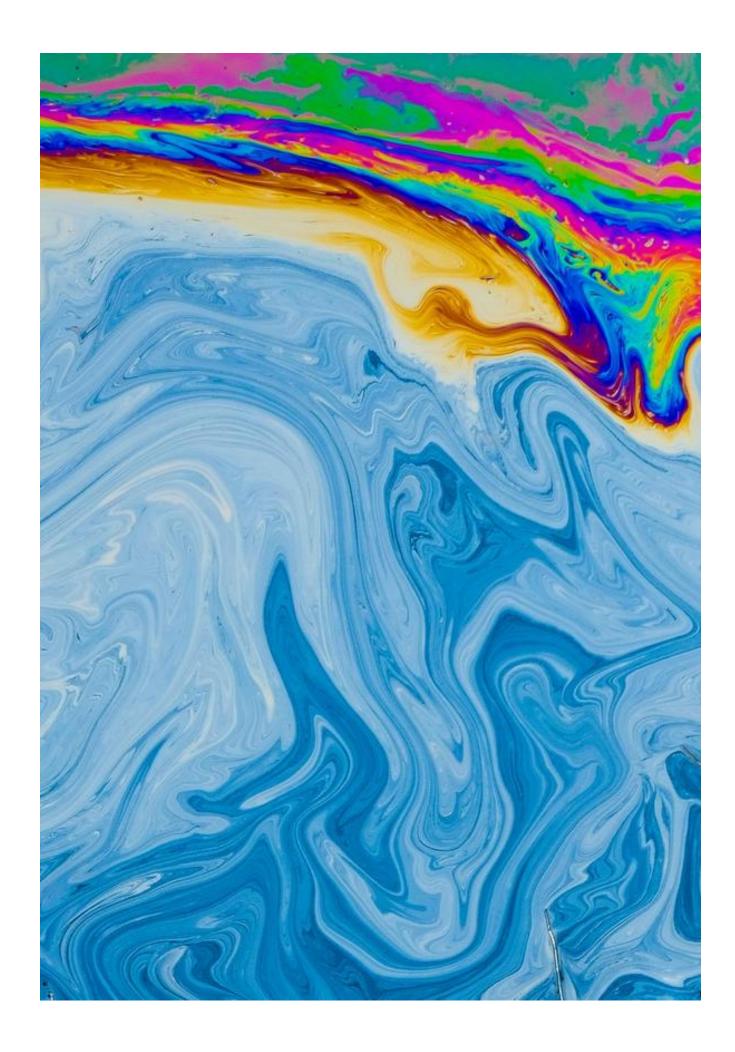
$$E_{data} =$$

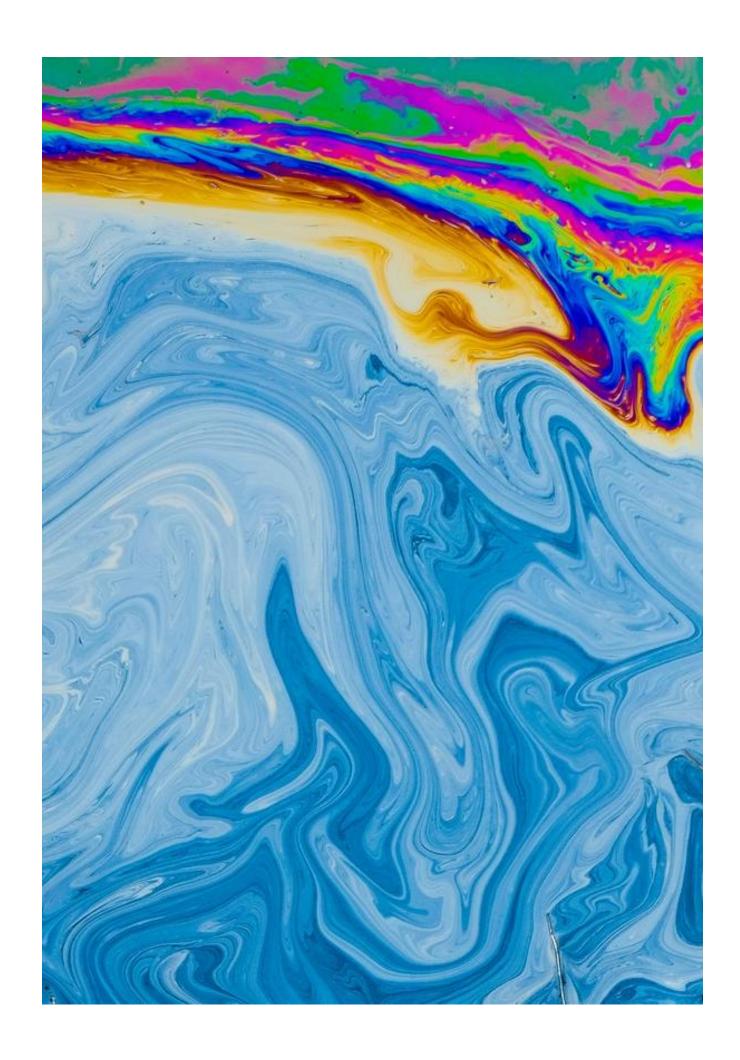
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  - $\lambda$  controls the clumpiness of the regions in the output.
  - We solve this problem with multi-label optimization [Boykov and Kolmogorov 2001].

$$\sum_{p \in I} \|R_p - I_p\|_2$$

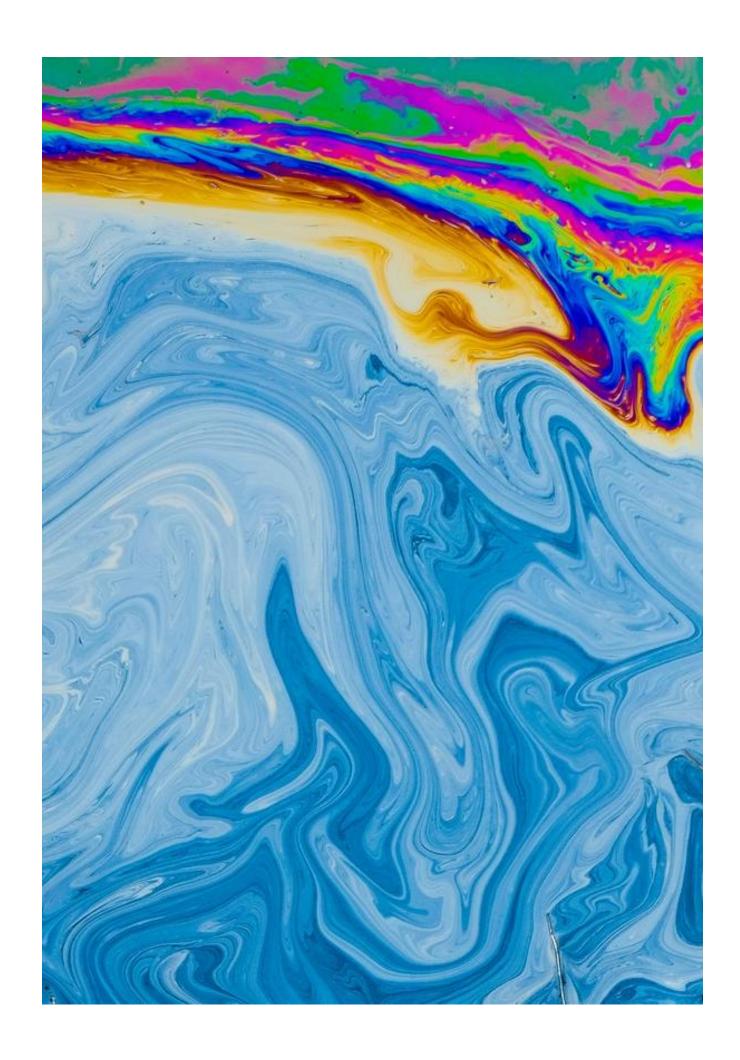
$$= \sum_{p,q \in N} \|L_p - L_q\|_2$$

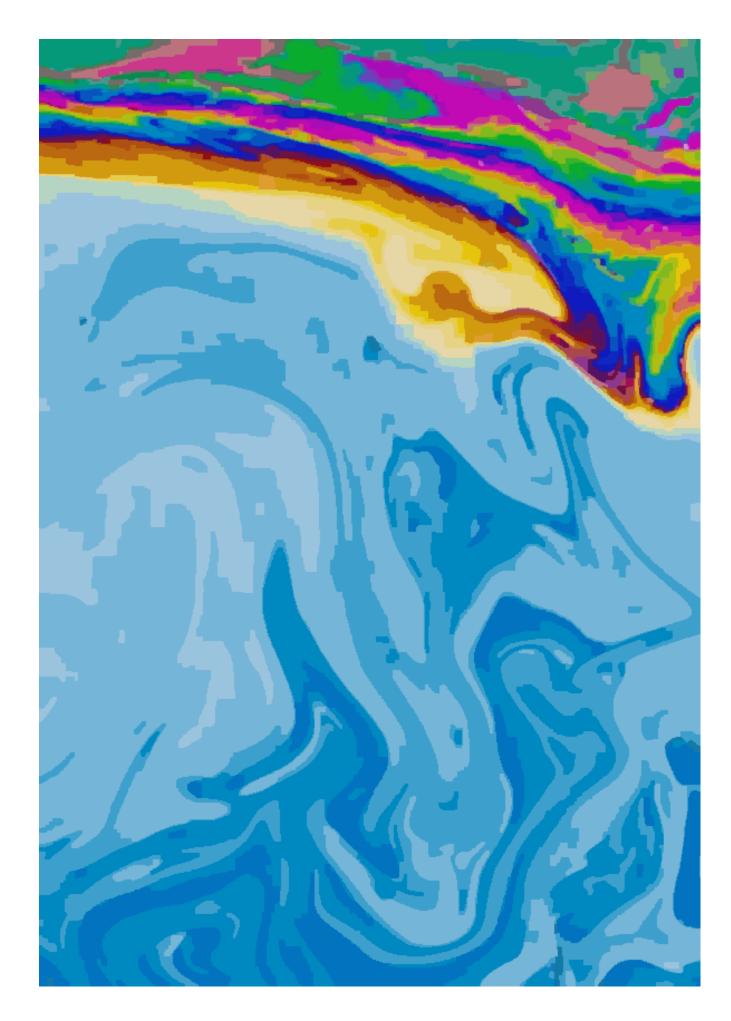




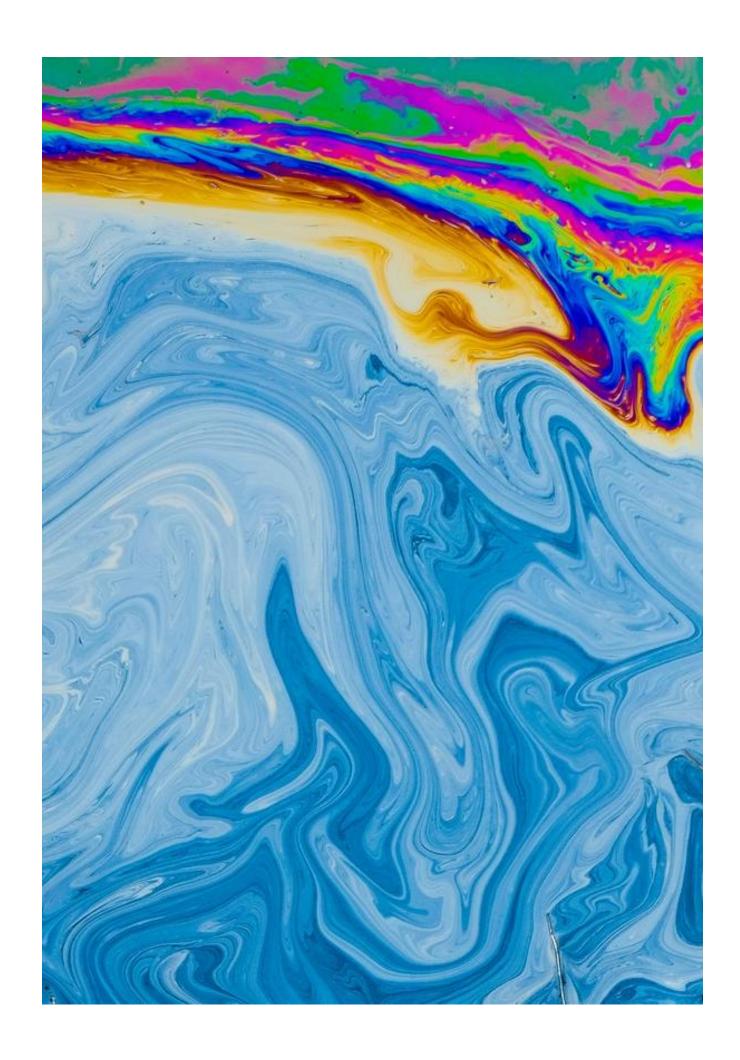


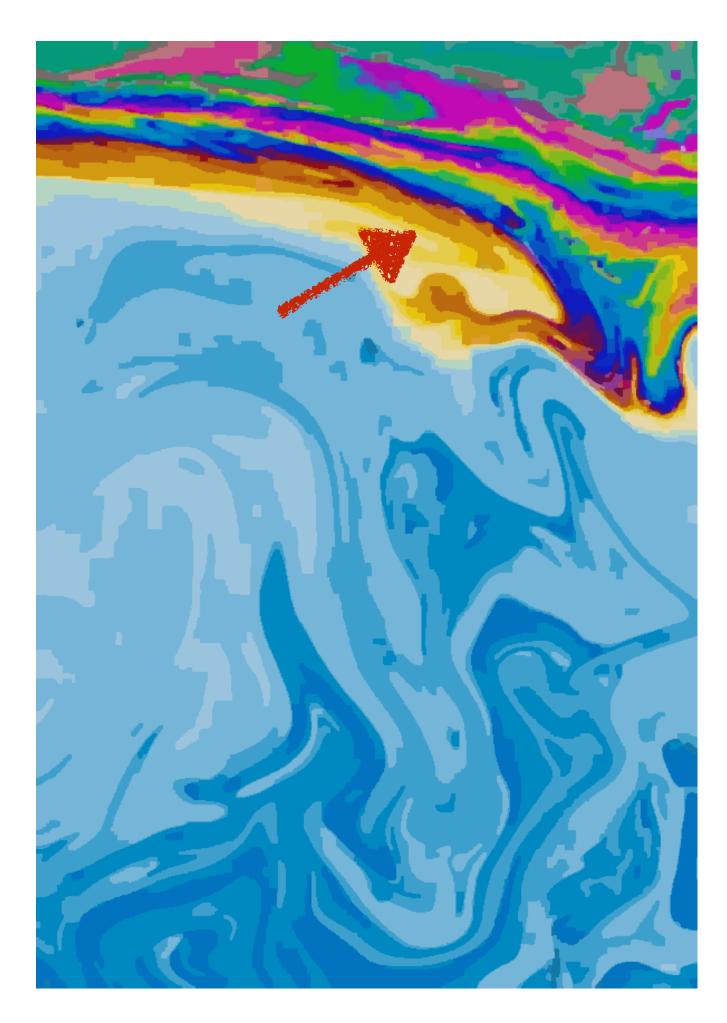
 $\min E(f)$ 



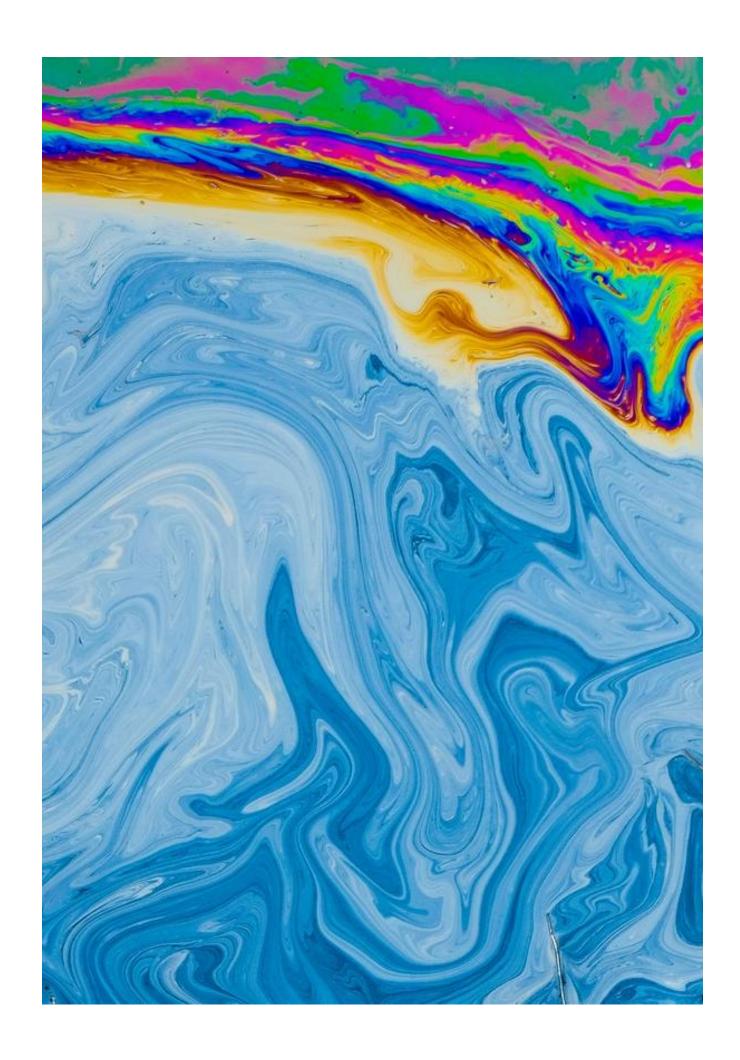


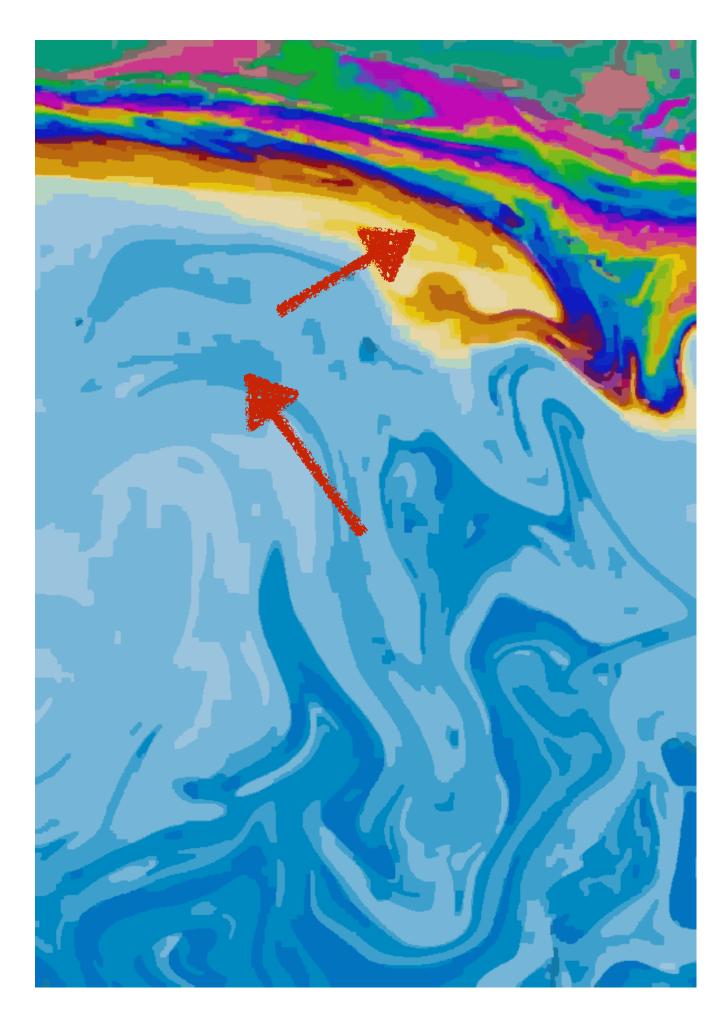
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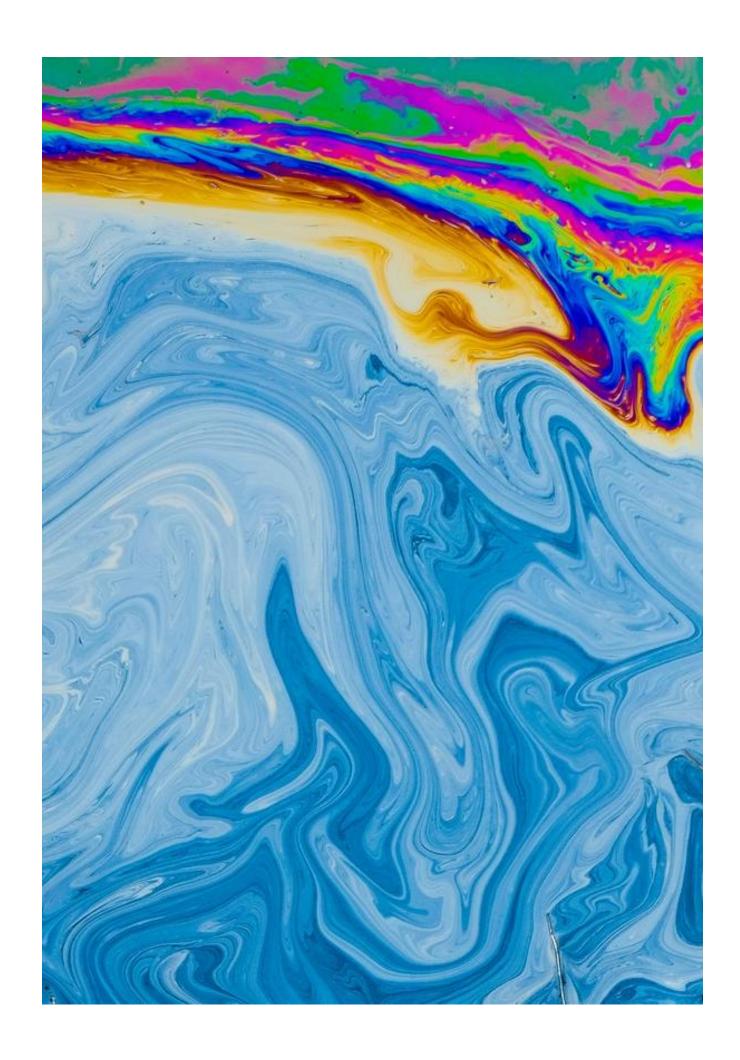


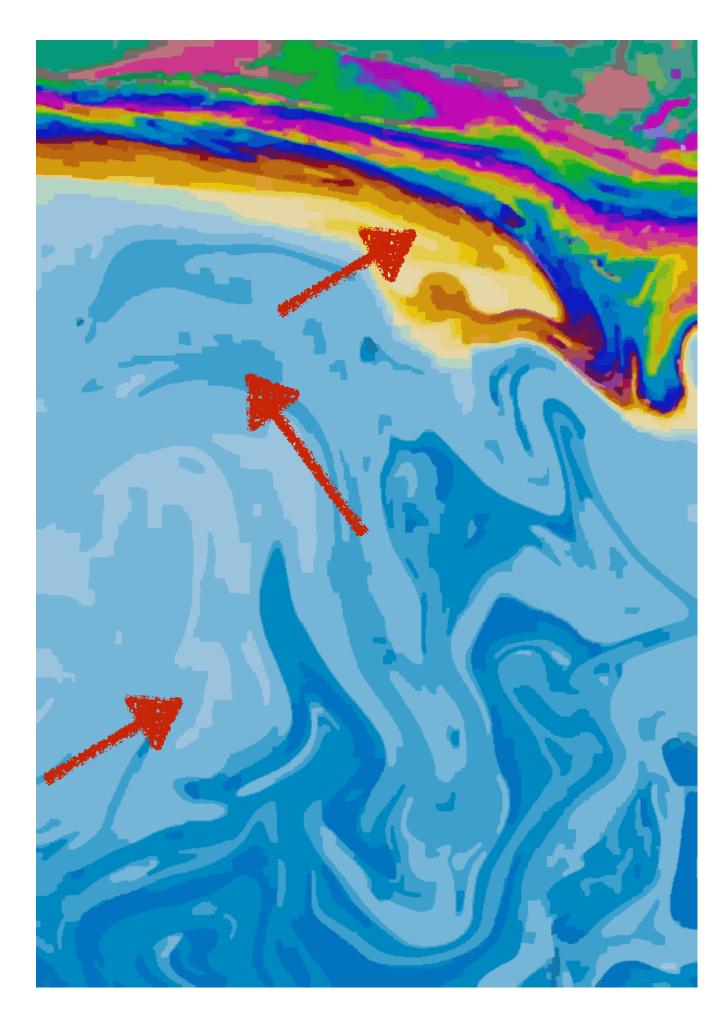
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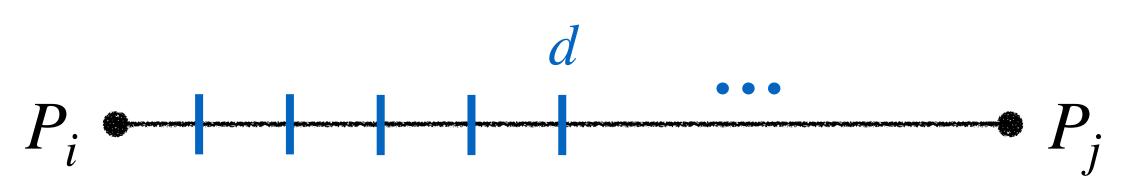


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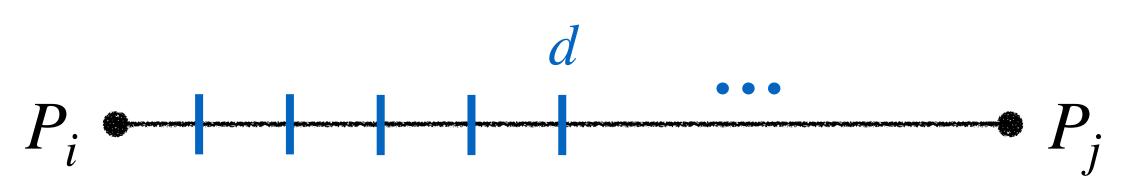
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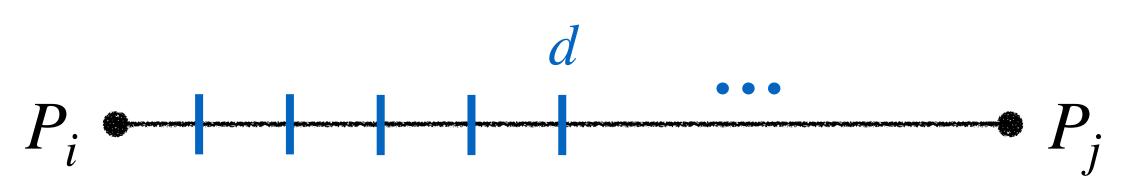


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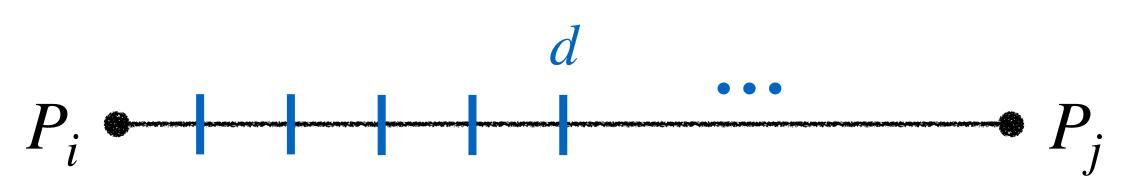
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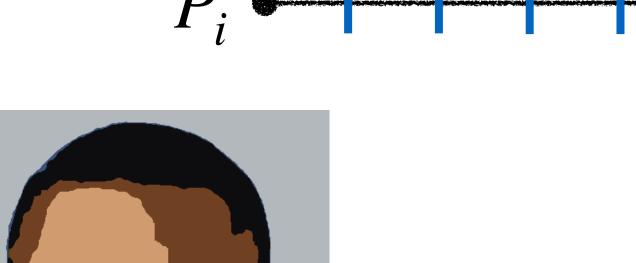
Input

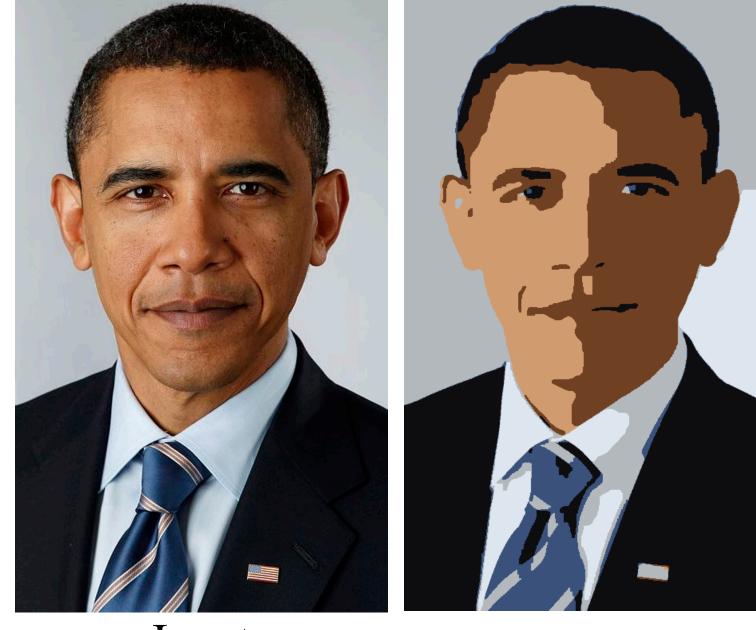
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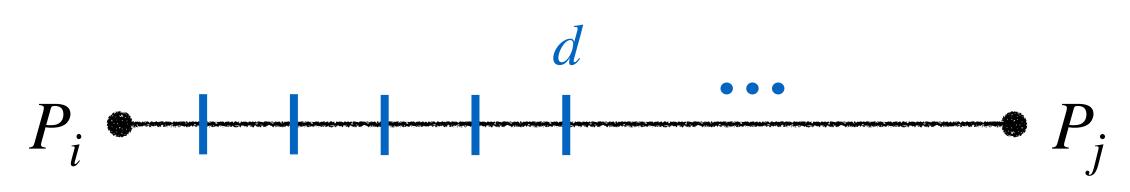


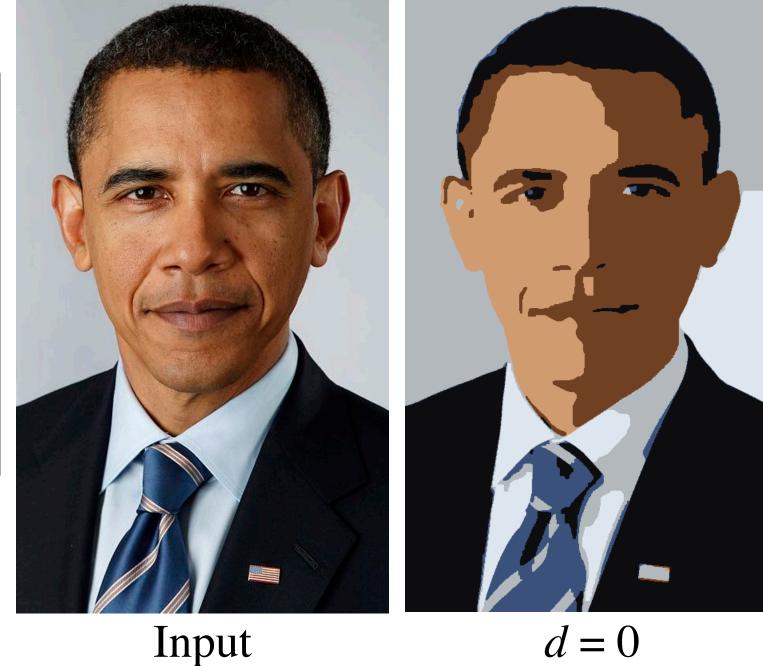


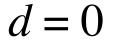
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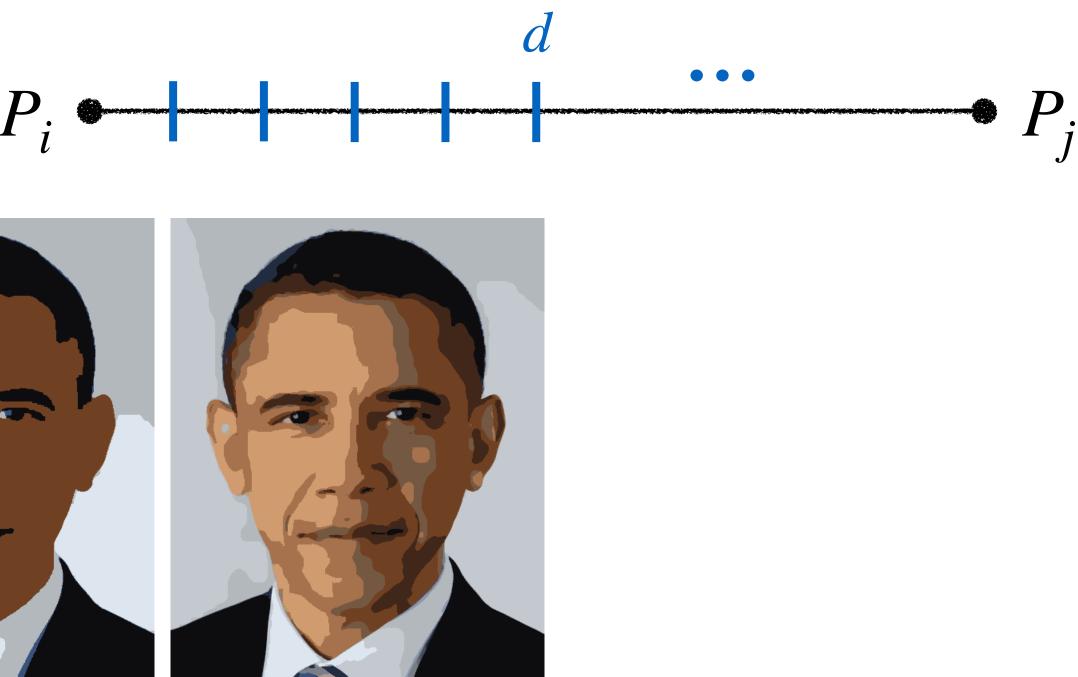
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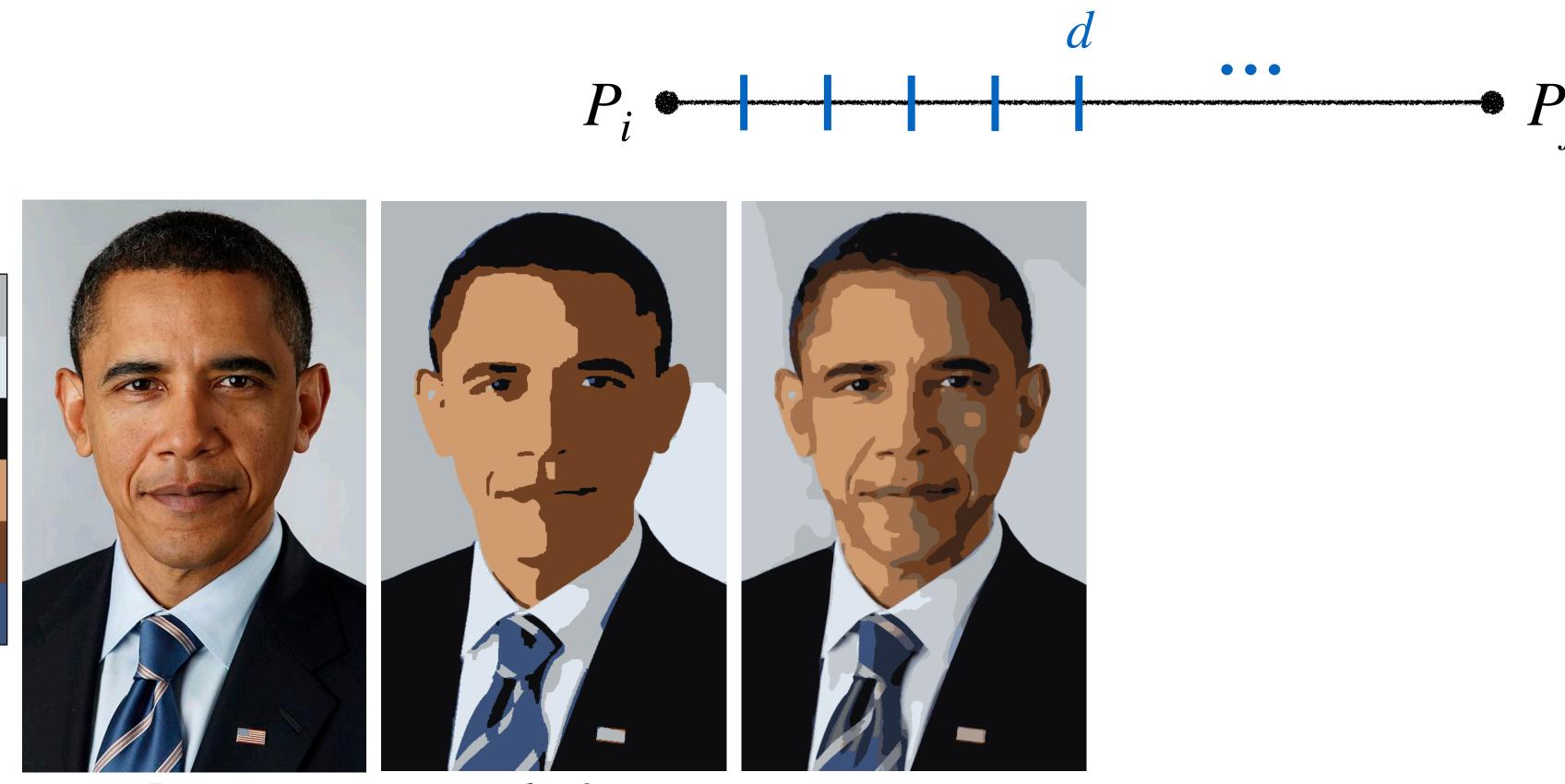






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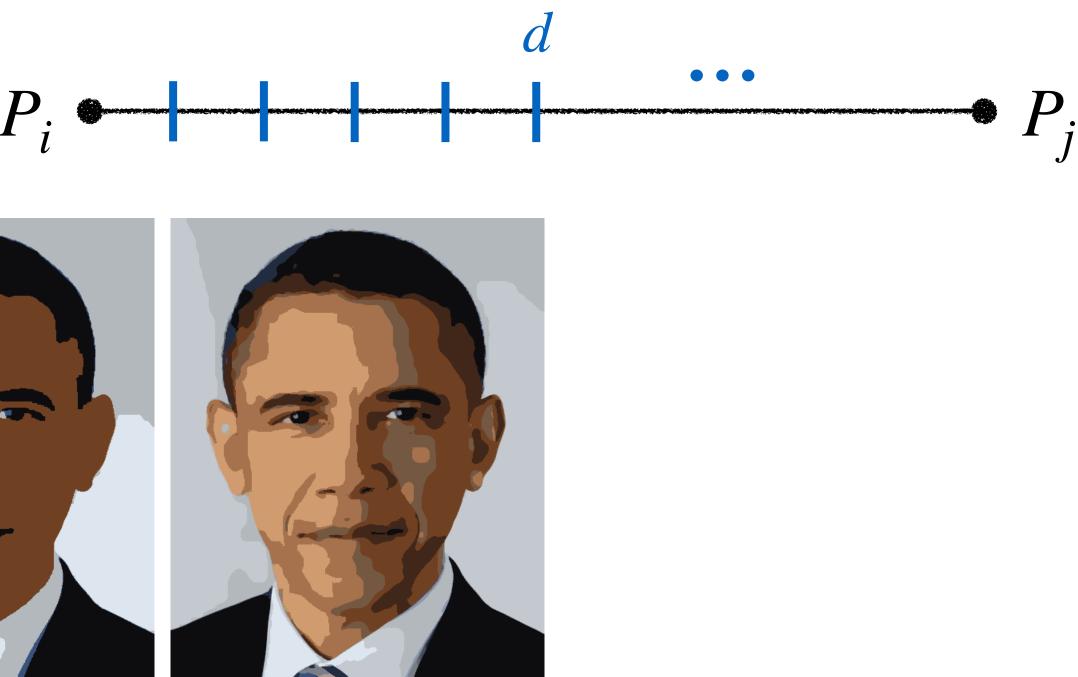


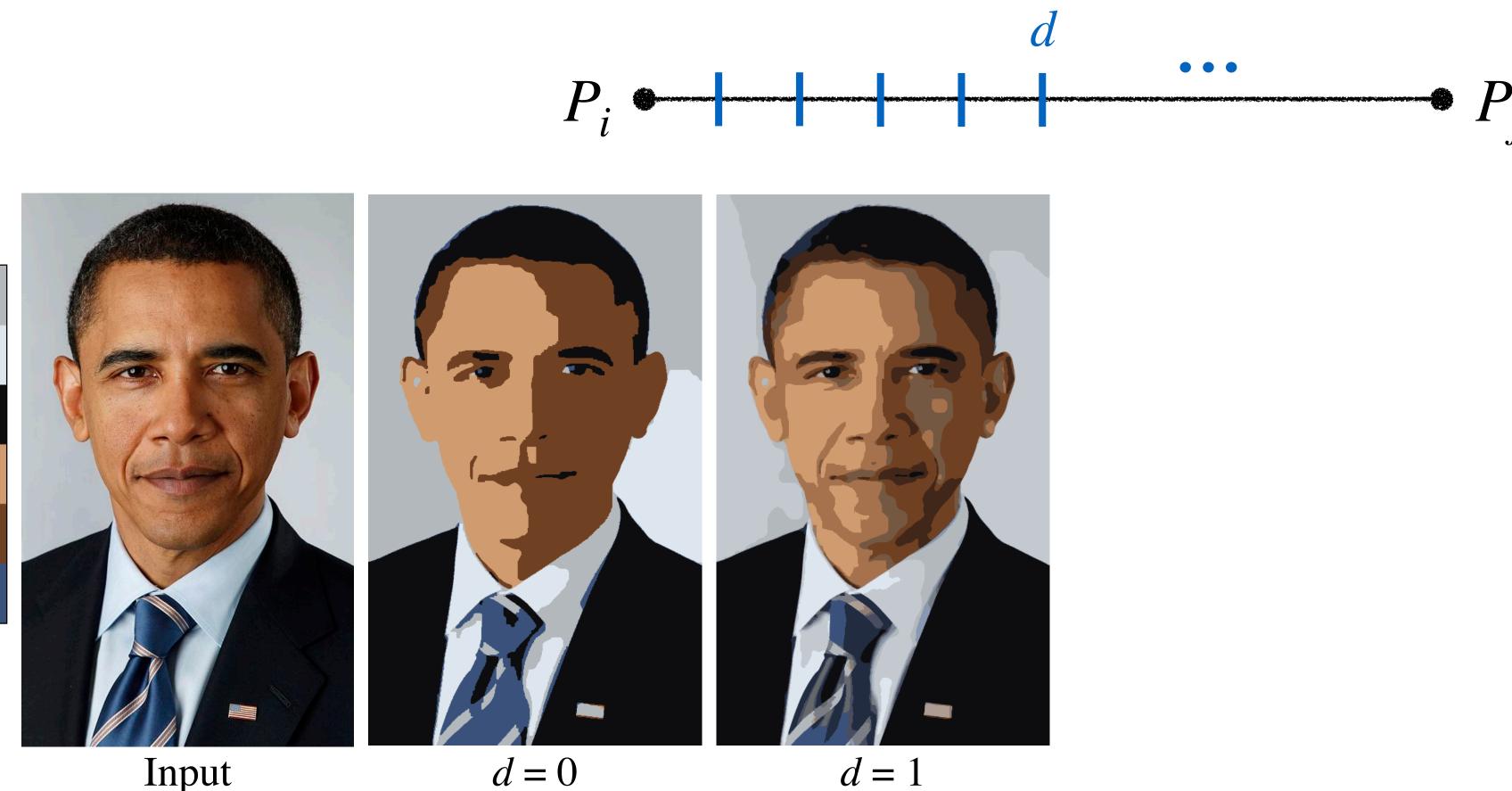


Input

d = 0

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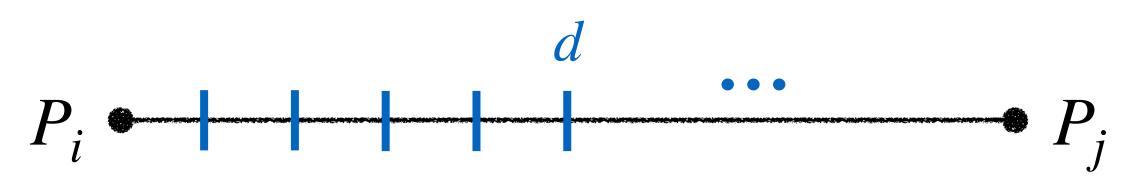


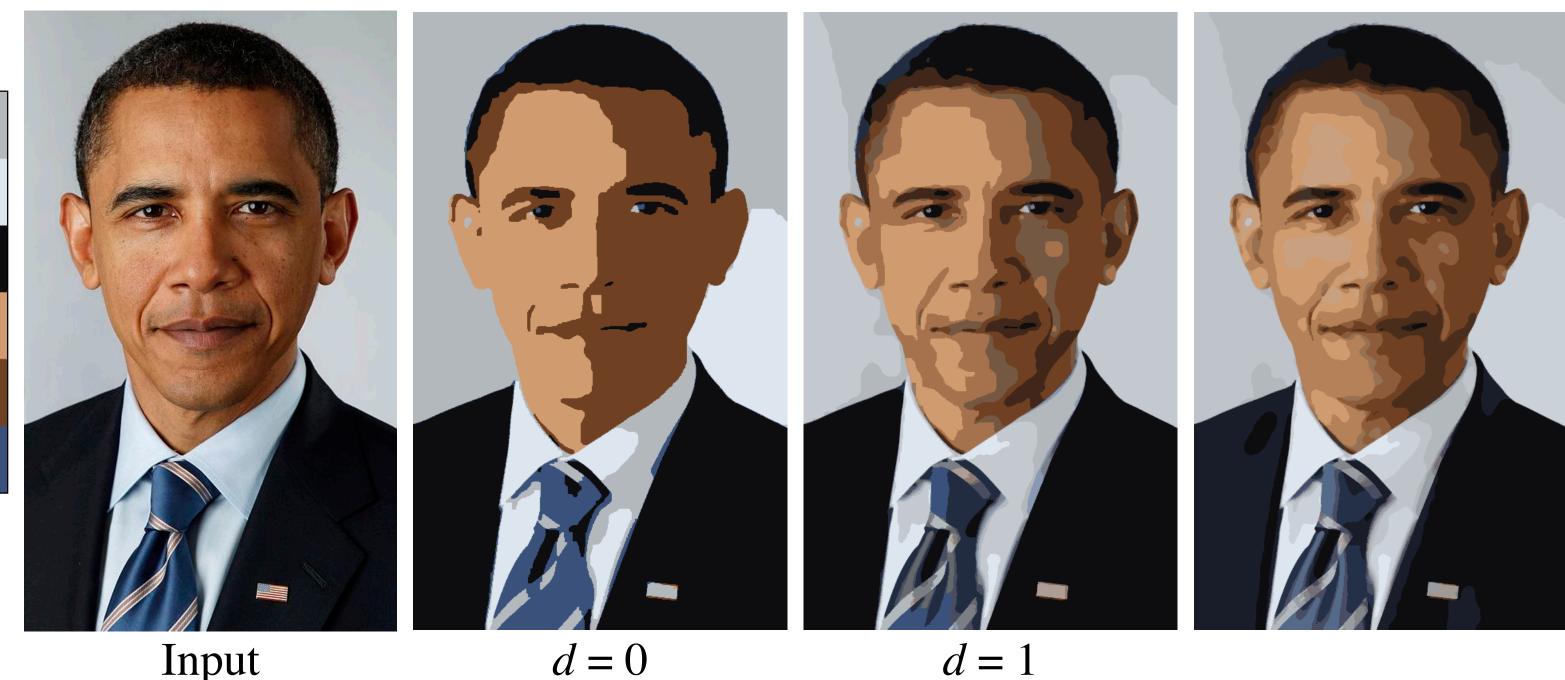


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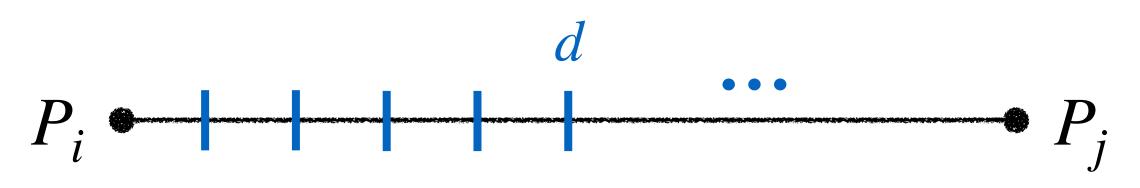


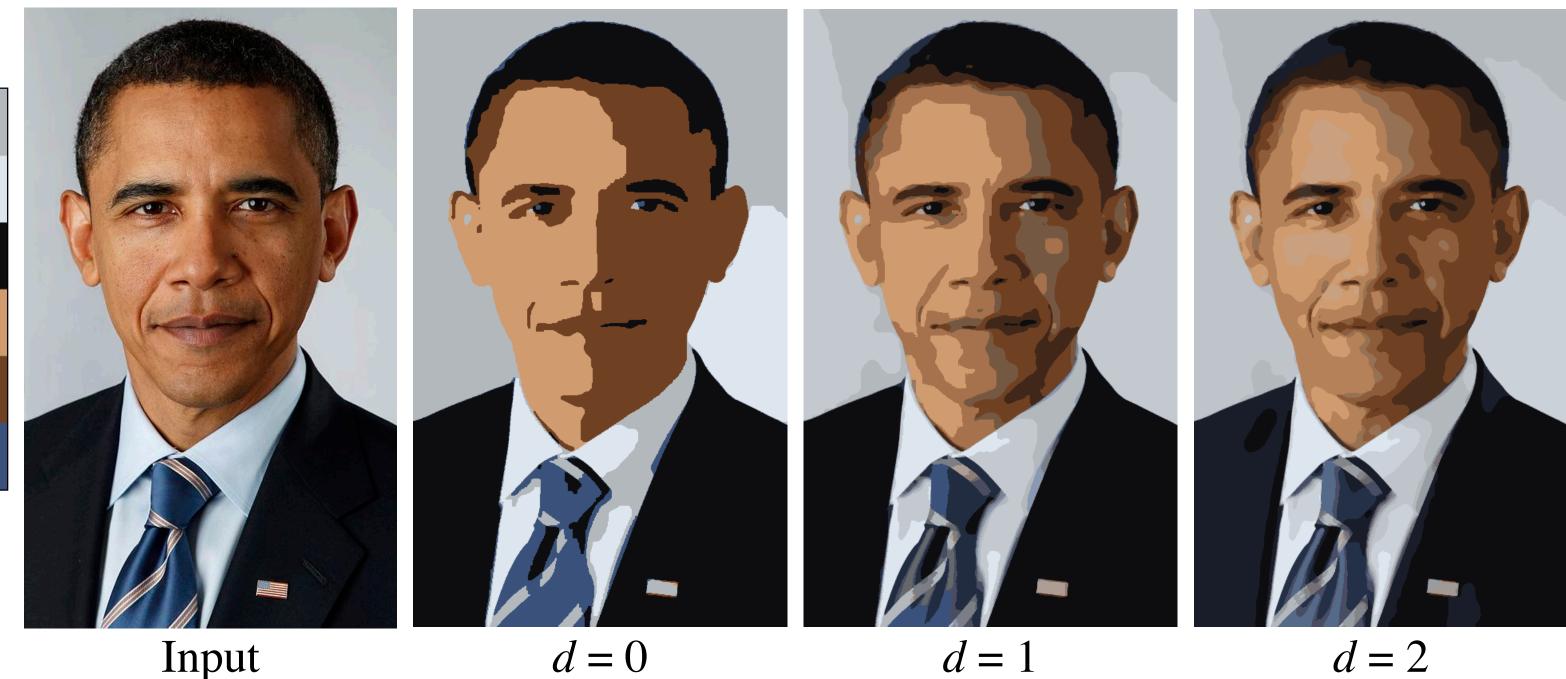


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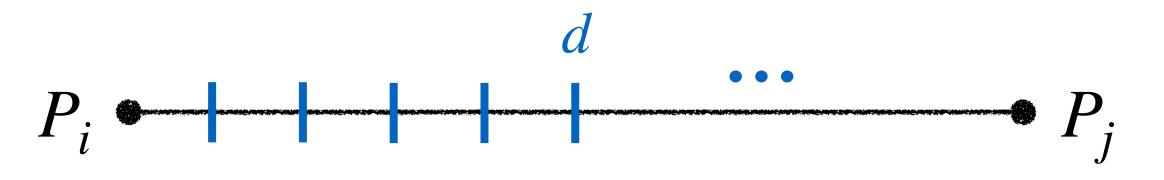


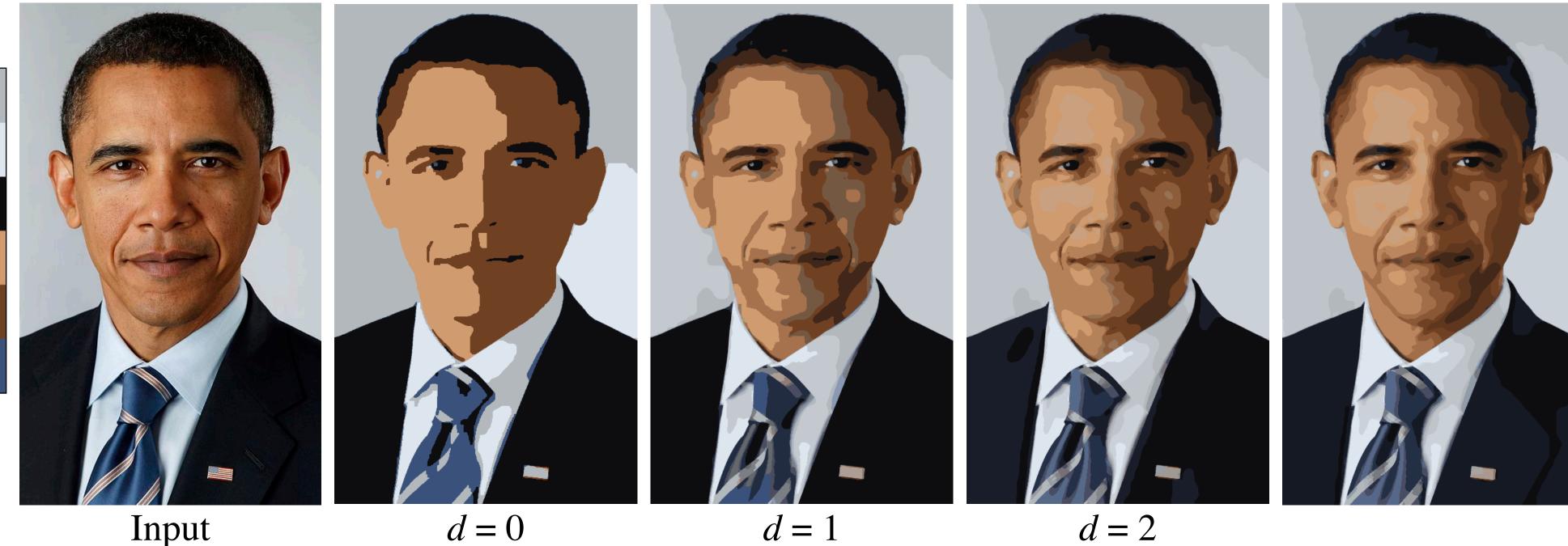


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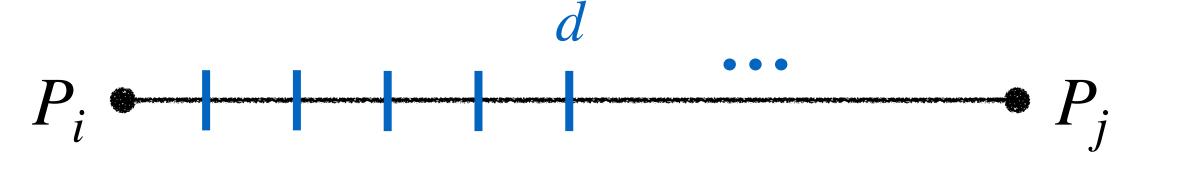


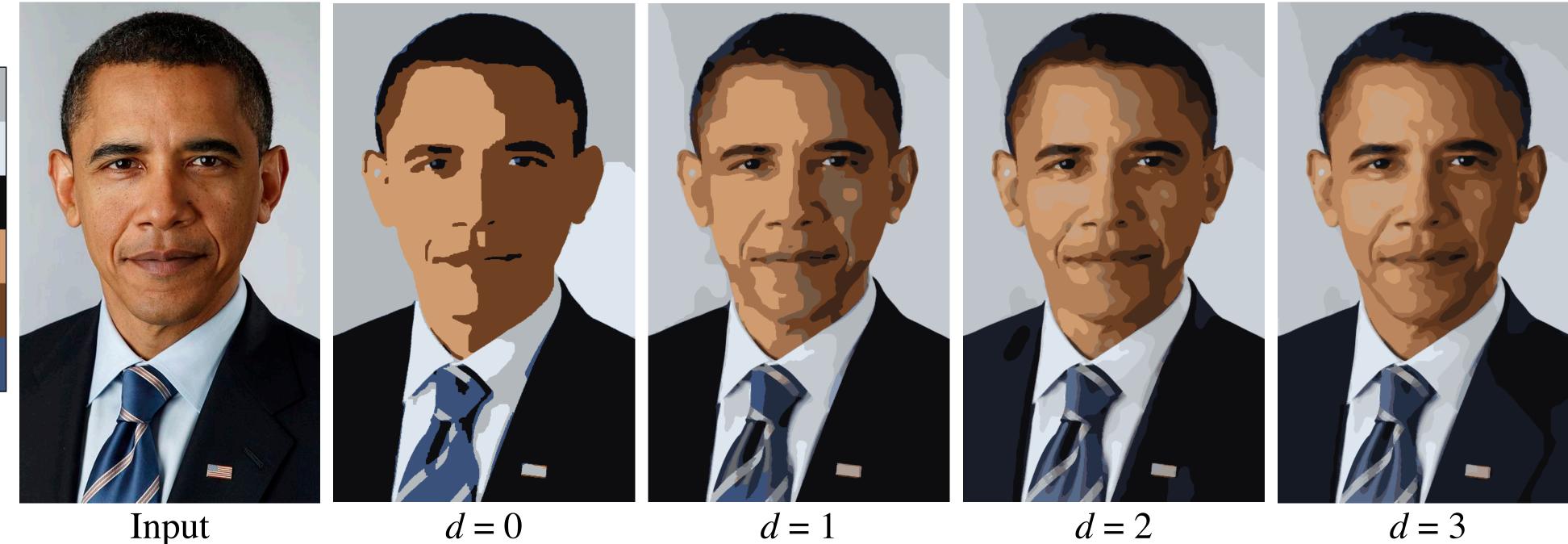
Input

d = 0

d = 2

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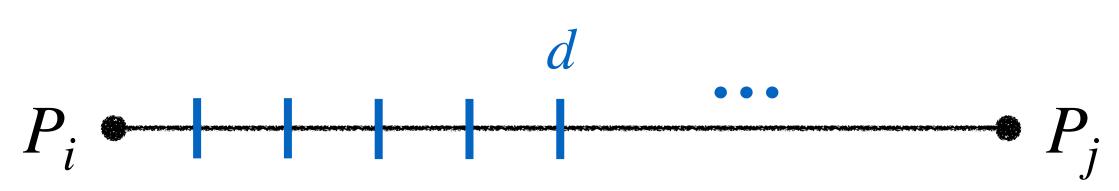


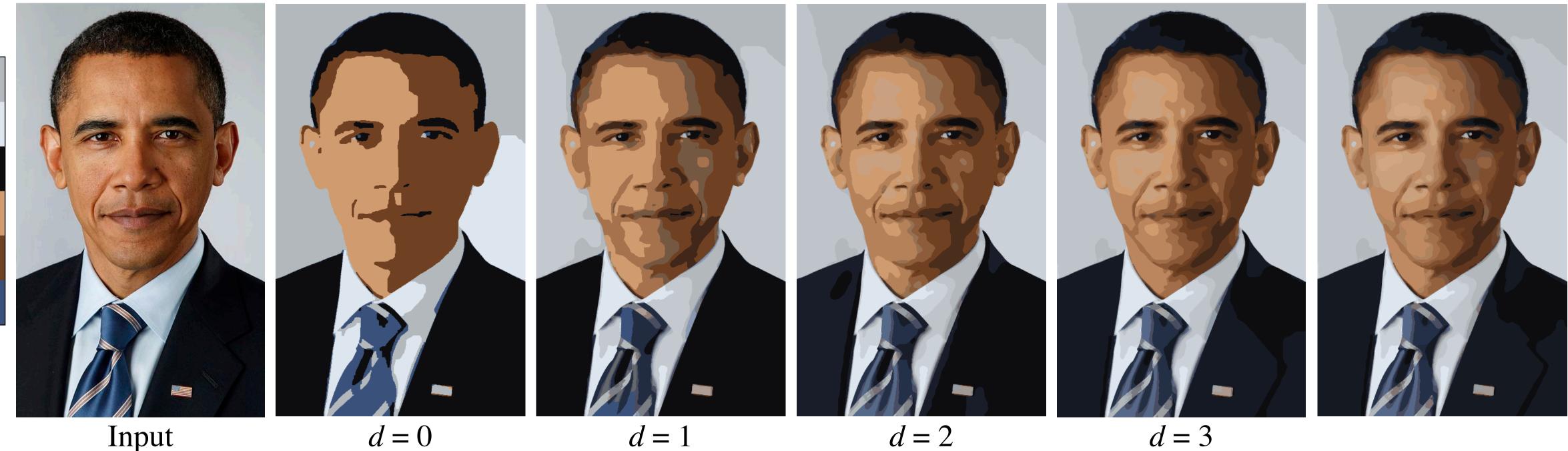


Input

d = 0

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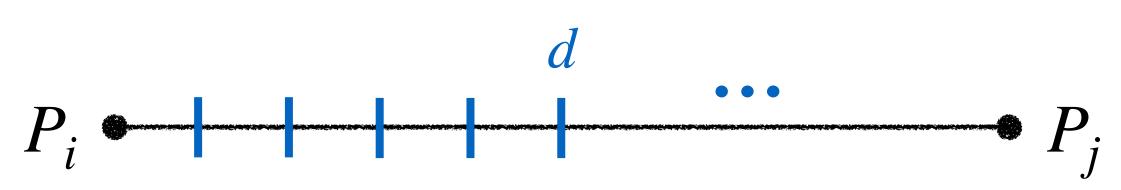
Input

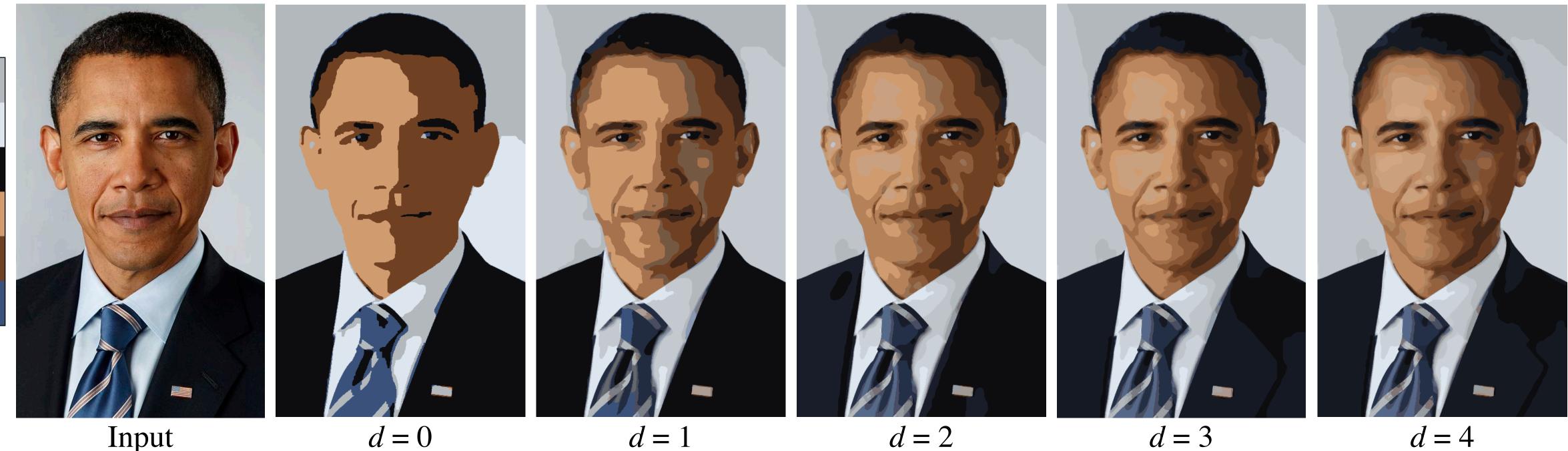
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d = 2

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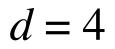


Input

d = 0

d = 2

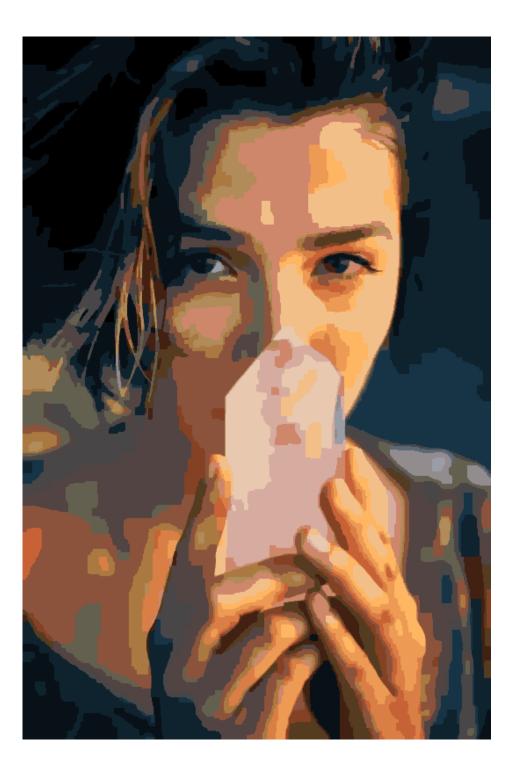
d = 3



- Step 3: Assign each region a continuous rather than discrete blend.
- See our paper for details.

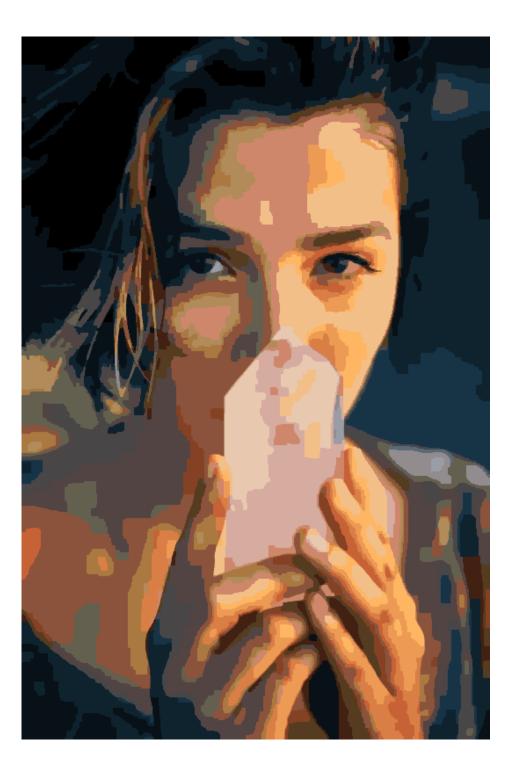
• Step 4: Smooth region boundaries with a frequency-guided median filter.

- Step 3: Assign each region a continuous rather than discrete blend.
- See our paper for details.



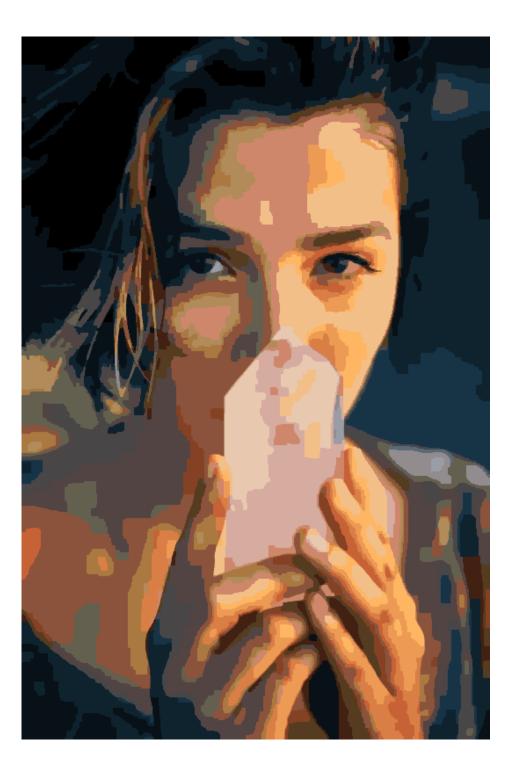
• Step 4: Smooth region boundaries with a frequency-guided median filter.

- Step 3: Assign each region a continuous rather than discrete blend.
- See our paper for details.



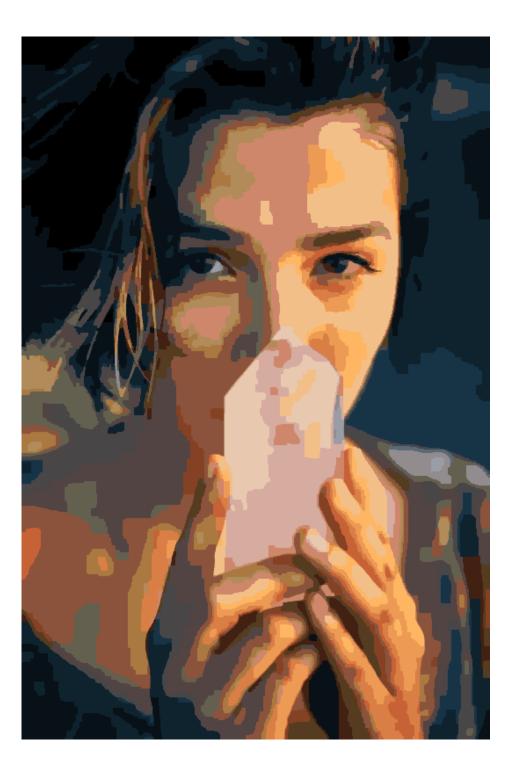
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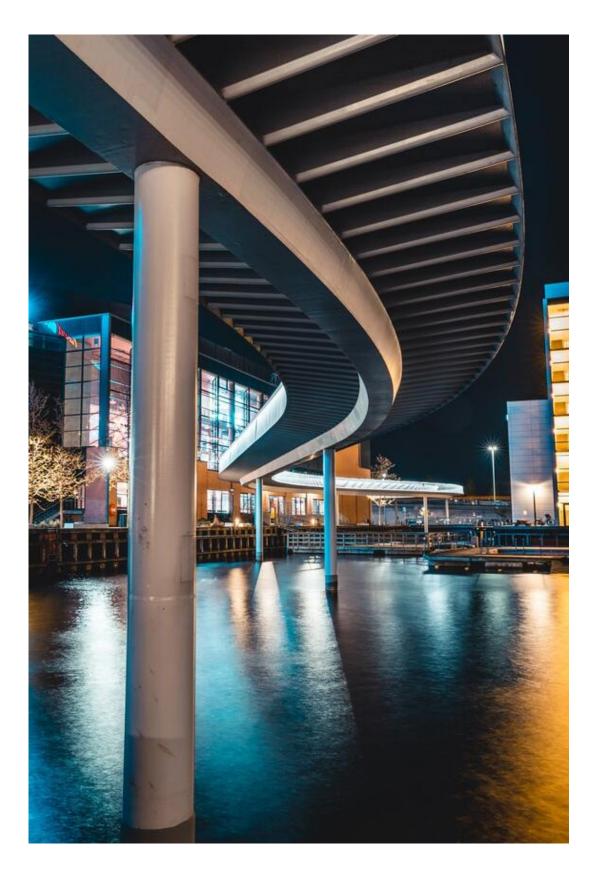


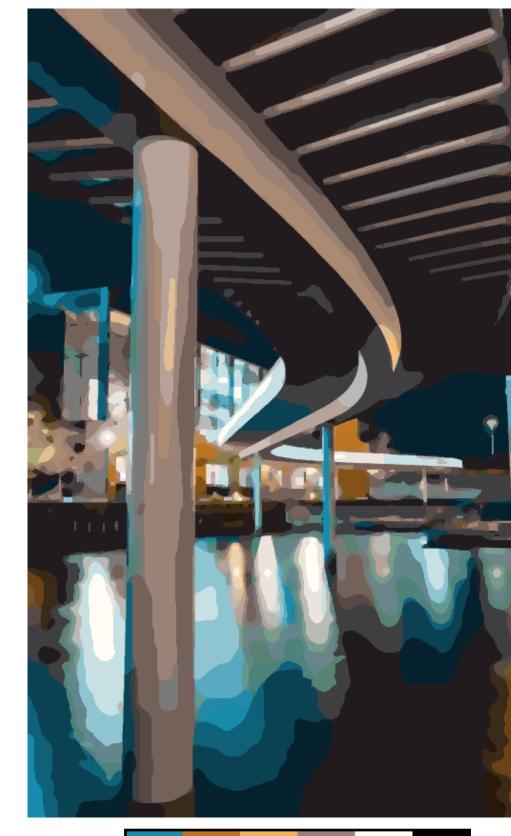
• Step 4: Smooth region boundaries with a frequency-guided median filter.

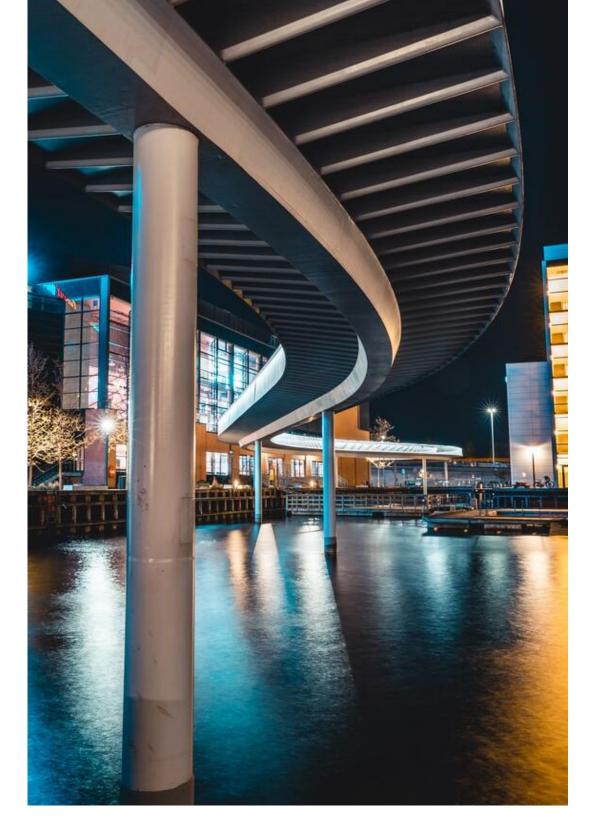




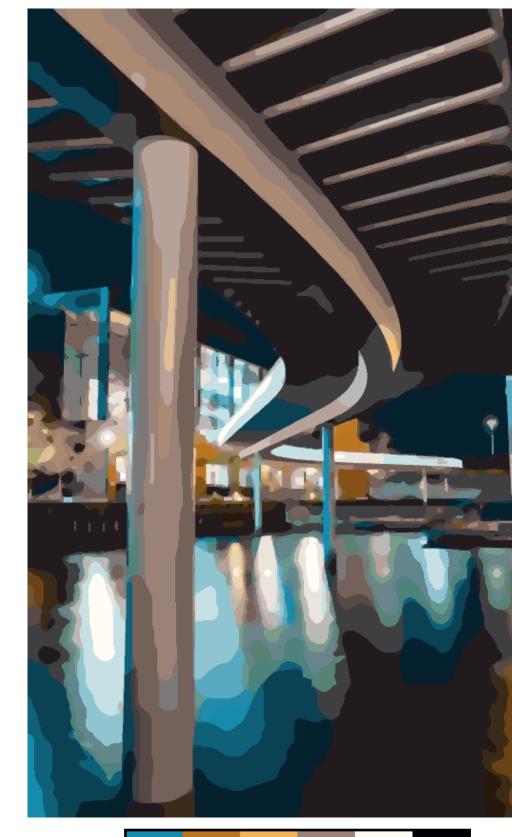


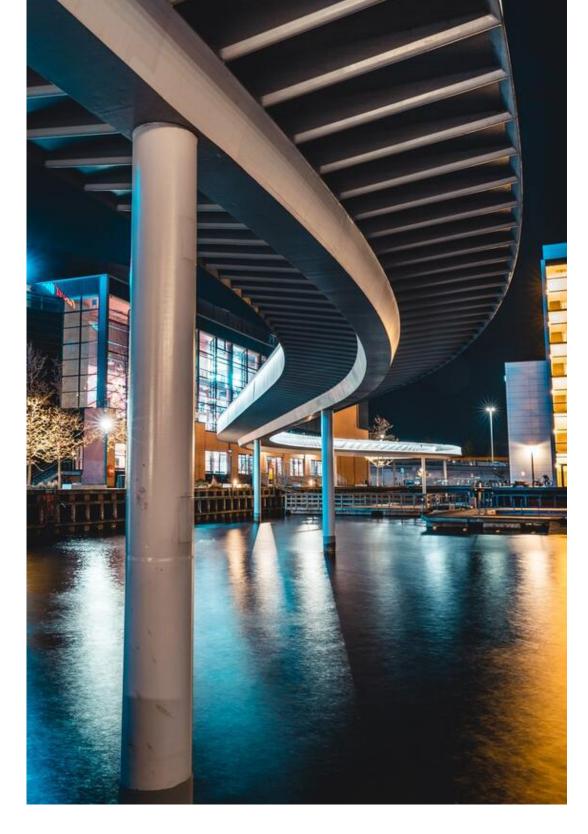






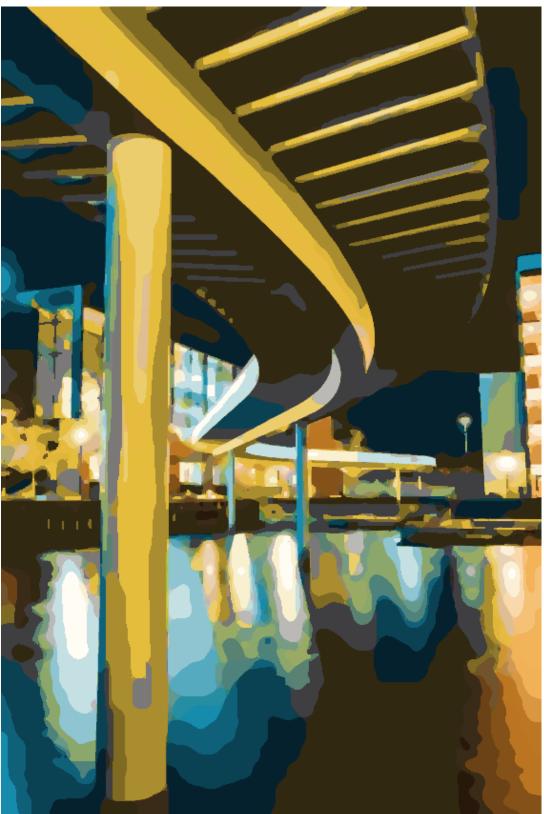




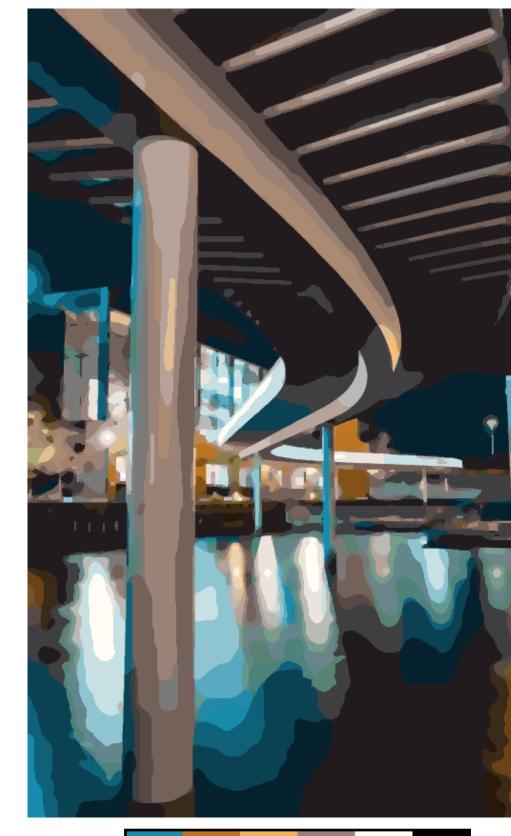


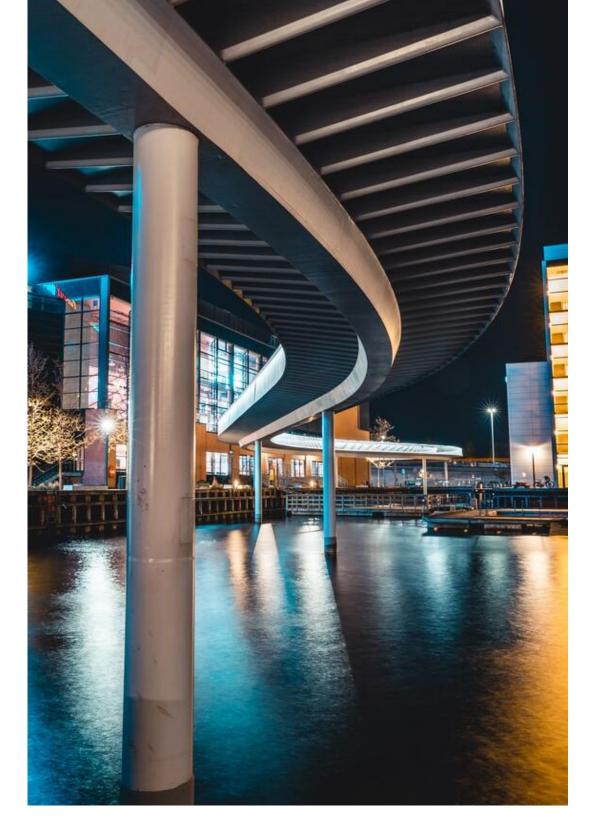








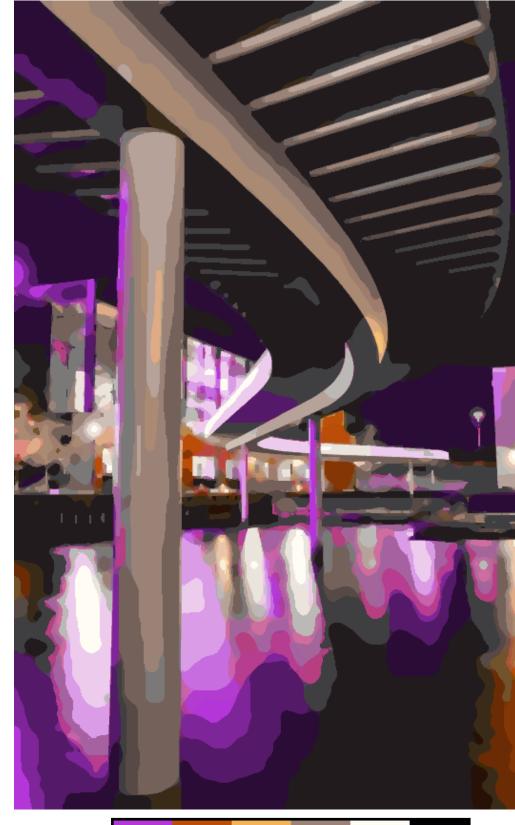


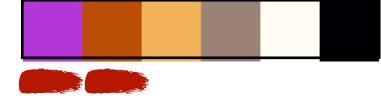




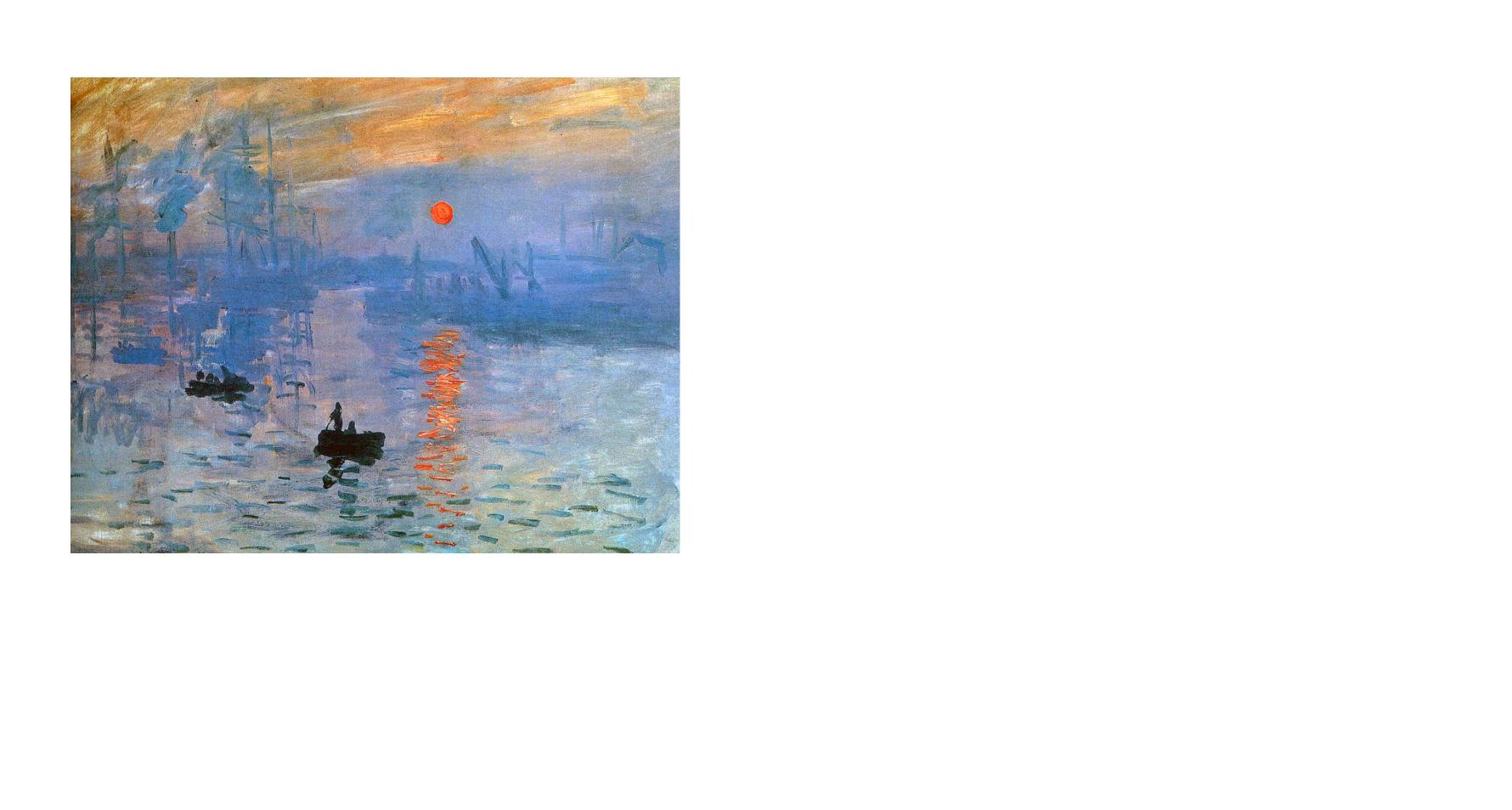


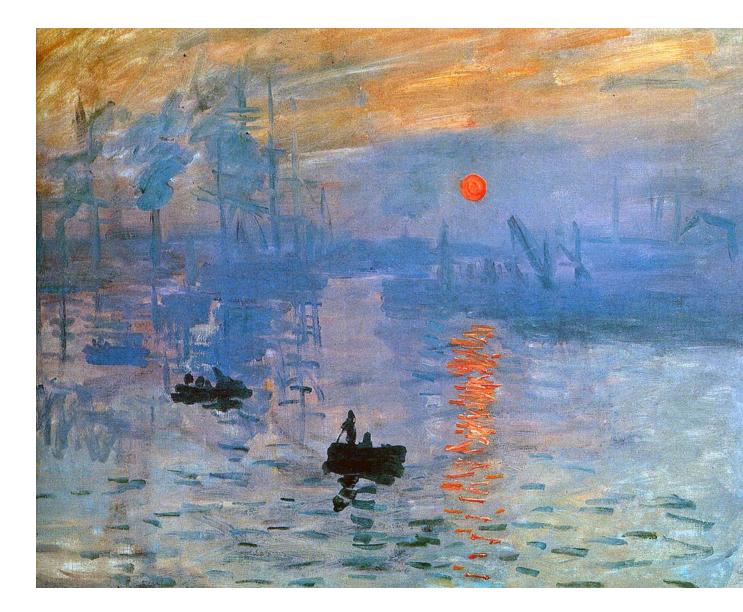


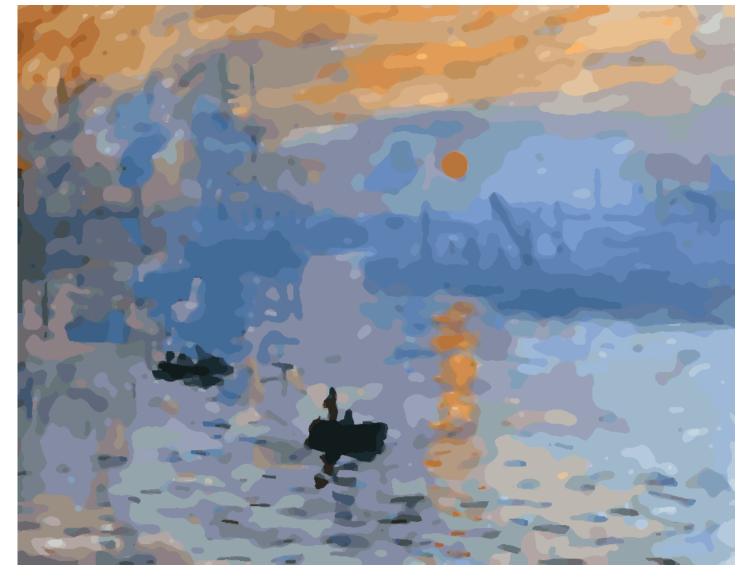




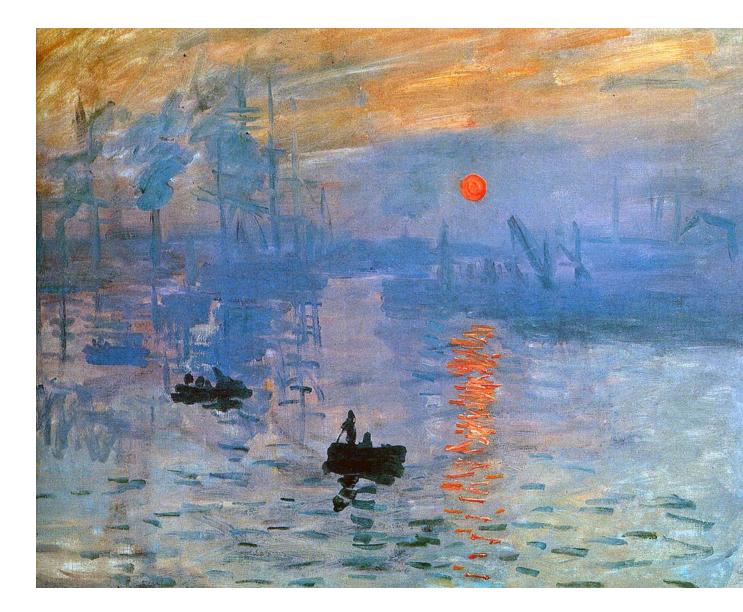






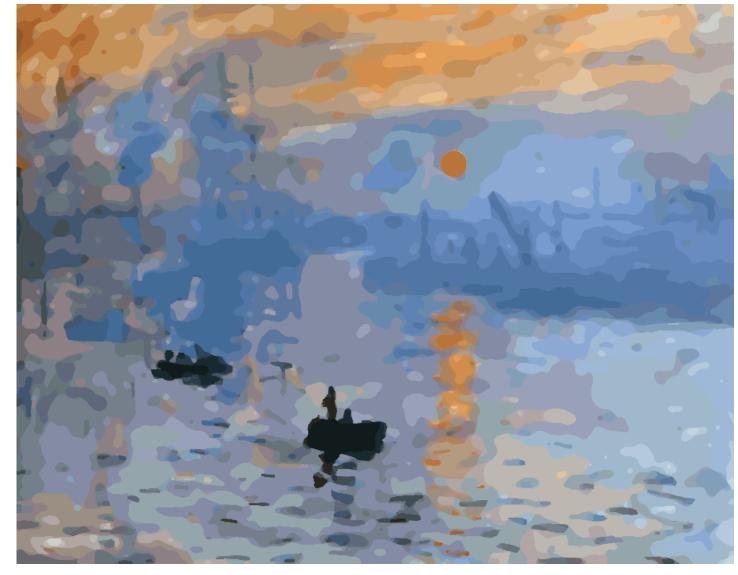






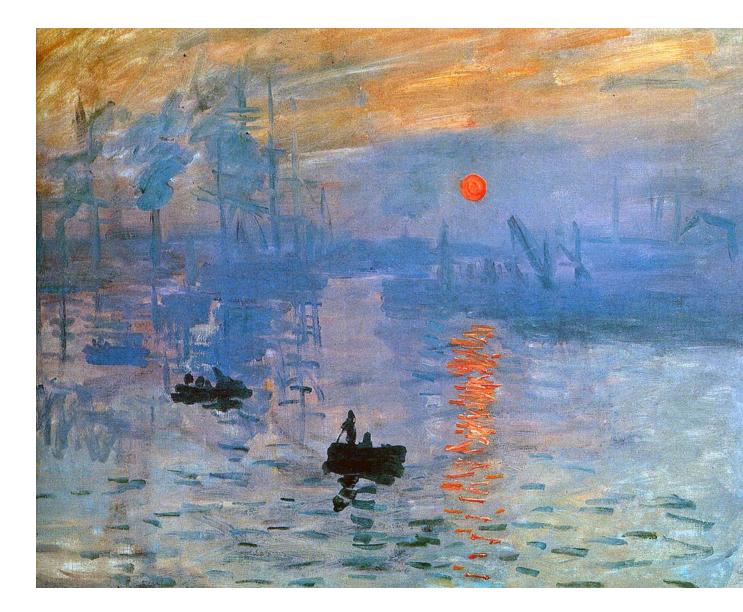


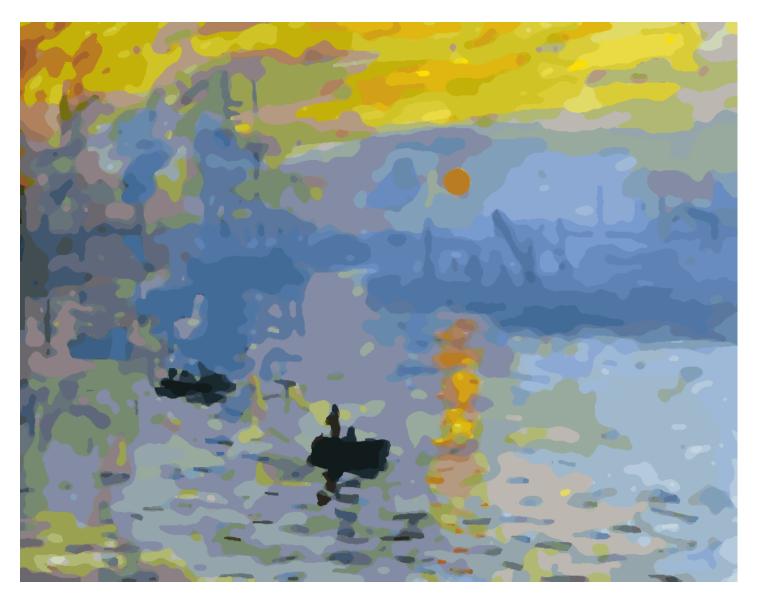






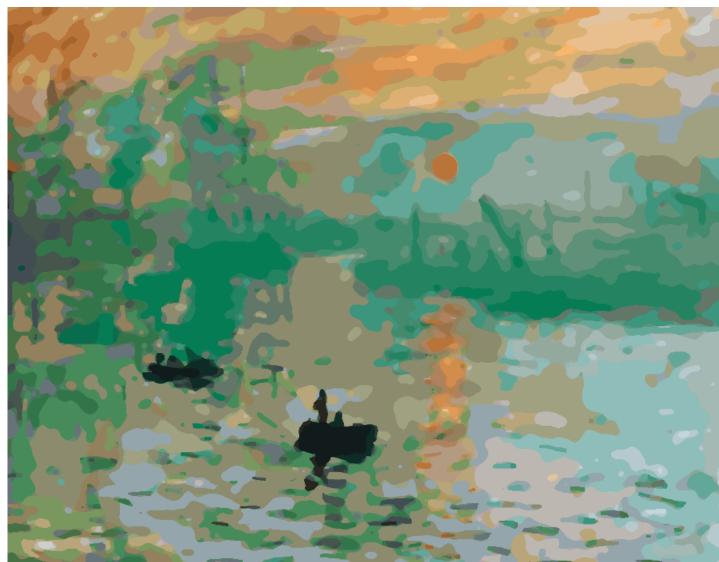


















#### Evaluation





#### Evaluation







Input

[Afifi 2018]

## Evaluation



Direct K-means (*K*=7 in RGB-space)



Direct K-means (*K*=44 in RGB-space)



Direct K-means (*K*=10 in RGBXY-space)

Ours









Input

[Afifi 2018]

[Xu and Kaplan 2008]

## Evaluation



Direct K-means (*K*=7 in RGB-space)



Direct K-means (*K*=44 in RGB-space)



Direct K-means (*K*=10 in RGBXY-space)

Ours









Input

[Afifi 2018]

[Xu and Kaplan 2008]

## Evaluation



Direct K-means (*K*=7 in RGB-space)



Direct K-means (*K*=44 in RGB-space)



Direct K-means (*K*=10 in RGBXY-space)

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Input

[Afifi 2018]

[Xu and Kaplan 2008]

## Evaluation



Direct K-means (*K*=7 in RGB-space)



Direct K-means (*K*=44 in RGB-space)



Direct K-means (*K*=10 in RGBXY-space)

Ours









Input

[Afifi 2018]

[Xu and Kaplan 2008]

## Evaluation

#### (44 colors)



Direct K-means (*K*=7 in RGB-space)



Direct K-means (*K*=44 in RGB-space)



Direct K-means (*K*=10 in RGBXY-space)



Ours







Input

[Afifi 2018]

[Xu and Kaplan 2008]

## Evaluation

#### (44 colors)



Direct K-means (*K*=7 in RGB-space)



Direct K-means (*K*=44 in RGB-space)



Direct K-means (*K*=10 in RGBXY-space)



Ours



- Comparison to related approaches.
- Expert study with professional artists.



Input

[Afifi 2018]

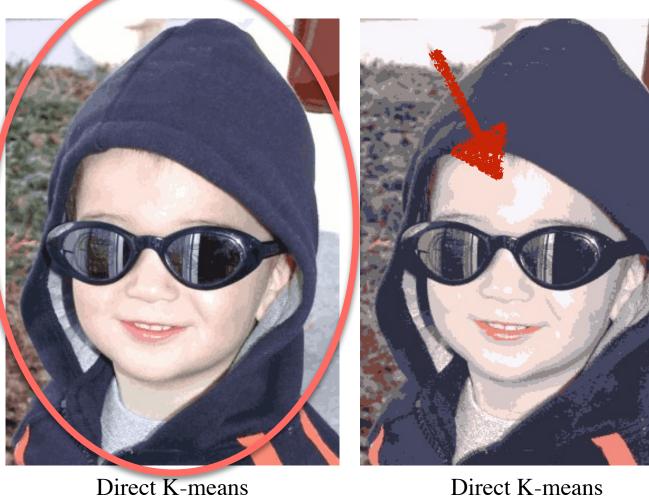
[Xu and Kaplan 2008]

## Evaluation

(44 colors)



Direct K-means (*K*=7 in RGB-space)



(*K*=44 in RGB-space)

Direct K-means (*K*=10 in RGBXY-space)



Ours



## Evaluation

- Comparison to related approaches.
- Expert study with professional artists.



Input

[Afifi 2018]

[Xu and Kaplan 2008]

•••		Posterization
Choose Image Save Current Image Save	Palette	<<< See History
Palette size (Default: 6):	6	
Palette blends (Default: 3):	3	
Rare color suppression (Default: 20):	20	
Region clumpiness (Default: 0.8):	0.8	
Posterize Reset		
Detail abstraction (Default: 0.1):	0.1 7	
Recolor the image via its palette: Choose color:		
R: 0		
G: 0		
B: 0		
Reset Current Color Reset All Colors		

(44 colors)



Direct K-means (*K*=7 in RGB-space)



Direct K-means (*K*=44 in RGB-space)



Direct K-means (*K*=10 in RGBXY-space)



Ours



## Evaluation

- Comparison to related approaches.
- Expert study with professional artists.
- See paper for the details.



Input

[Afifi 2018]

[Xu and Kaplan 2008]

•••		Posterization
Choose Image Save Current Image Save	Palette	<<< See History
Palette size (Default: 6):	6	
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(44 colors)



Direct K-means (*K*=7 in RGB-space)



Direct K-means (*K*=44 in RGB-space)



Direct K-means (*K*=10 in RGBXY-space)



Ours

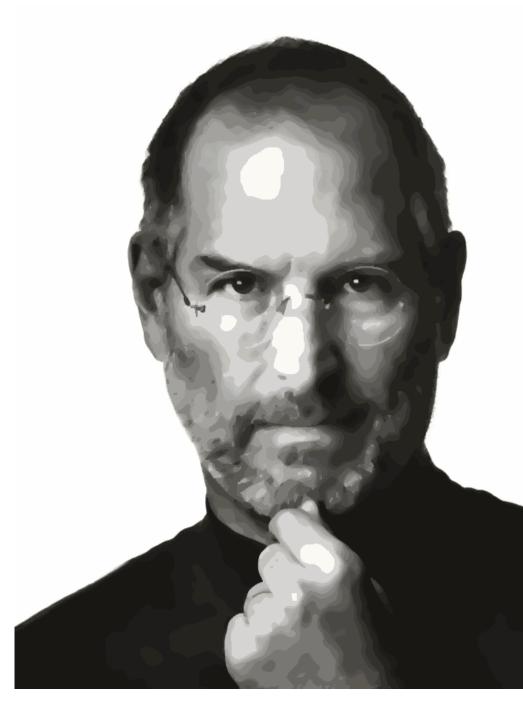




• PosterChild shows:

- PosterChild shows:
  - Qualitatively similar to those created by artists in a time-consuming manner.

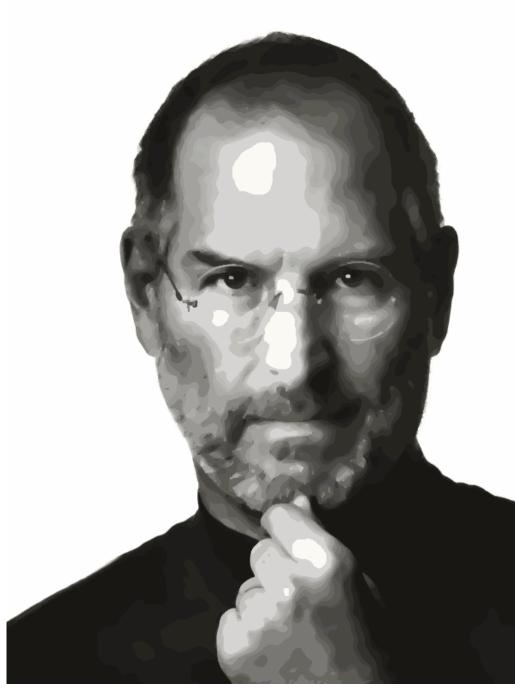
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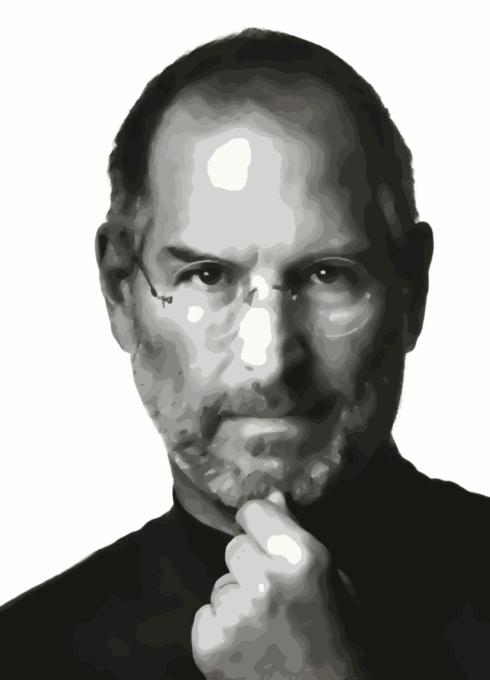
- PosterChild shows:
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  - Easy to do palette-based recoloring on posters in real-time.

artists in a time-consuming manner. posters in real-time.



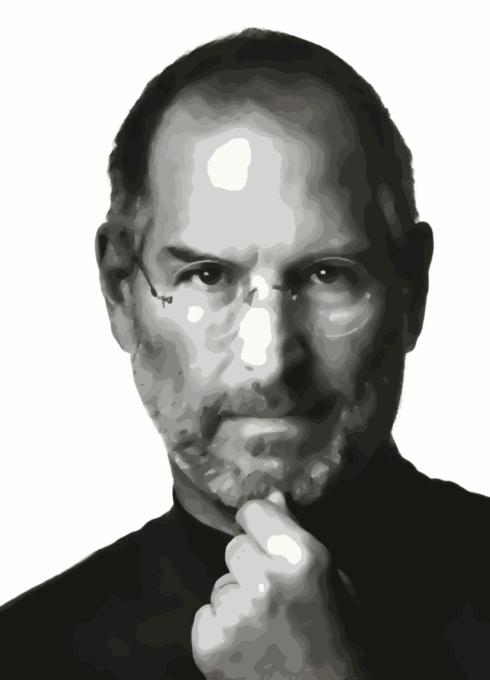


- PosterChild shows:
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  - Easy to do palette-based recoloring on posters in real-time.
  - Aesthetically outperform state-of-the-art automatic posterization tools.



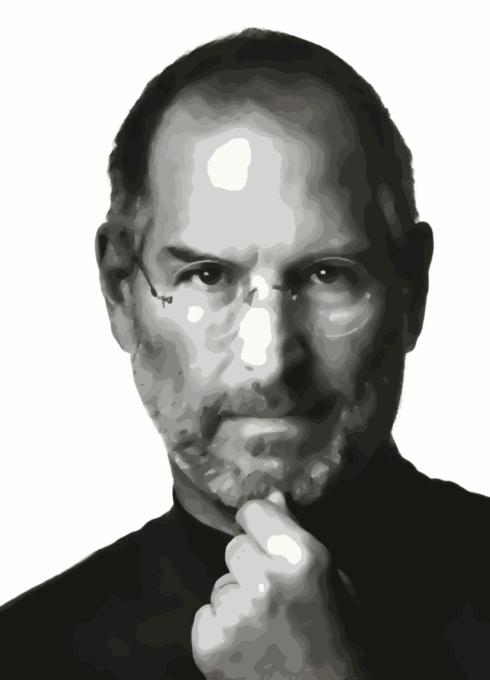


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  - Easy to do palette-based recoloring on posters in real-time.
  - Aesthetically outperform state-of-the-art automatic posterization tools.
- Limitations:



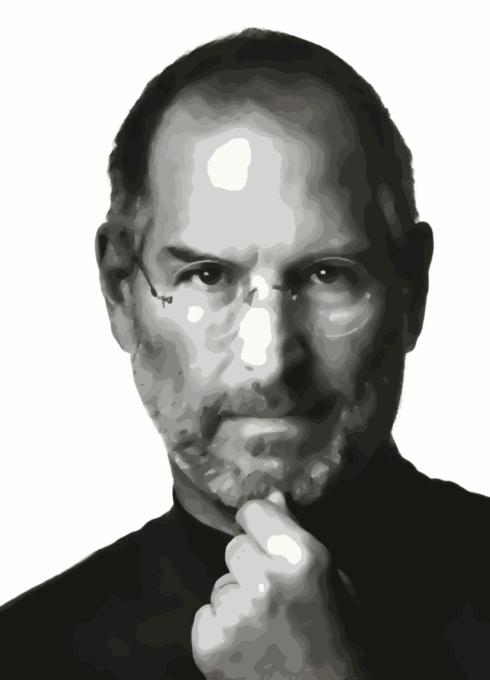


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- Limitations:
  - Only allows real-time recoloring.



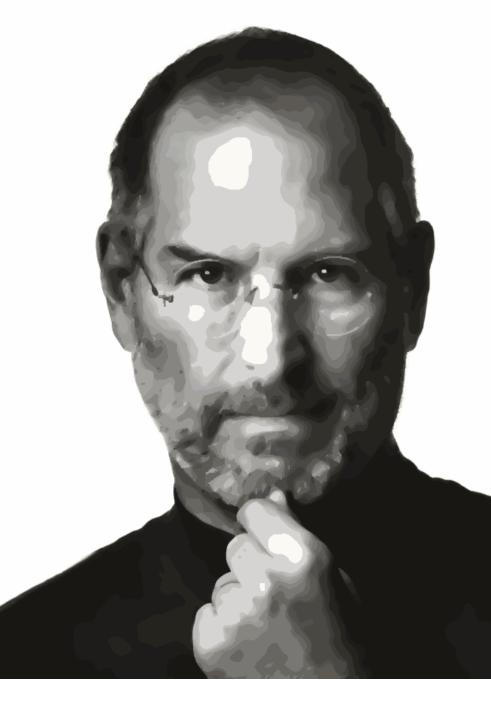


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- Limitations:
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  - Slow performance on outlier removal.





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  - Easy to do palette-based recoloring on posters in real-time.
  - Aesthetically outperform state-of-the-art automatic posterization tools.
- Limitations:
  - Only allows real-time recoloring.
  - Slow performance on outlier removal.
  - Does not recognize the semantics of input images.





- Code and GUI will be available at: https://cragl.cs.gmu.edu/
- Financial support
  - NSERC



# Thank You





- Code and GUI will be available at: https://cragl.cs.gmu.edu/
- Financial support
  - NSERC





# Thank You







• [Wang et al. 2019] observed that co outliers.

• [Wang et al. 2019] observed that convex-hull based palettes are sensitive to



- outliers.
- K-means as a relaxation on the input RGB colors.

• [Wang et al. 2019] observed that convex-hull based palettes are sensitive to







Input

Direct K-means clustering result







-

Posterized image (K-means clustering to eliminate outliers) Posterized image (without K-means clustering to eliminate outliers)











Input

Direct K-means clustering result



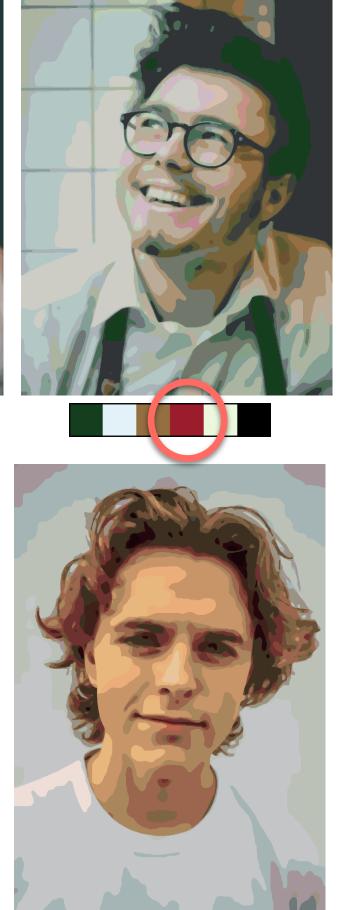




-

Posterized image (K-means clustering to eliminate outliers) Posterized image (without K-means clustering to eliminate outliers)









Input

Direct K-means clustering result







Posterized image (K-means clustering to eliminate outliers)

Posterized image (without K-means clustering to eliminate outliers)











Input

Direct K-means clustering result



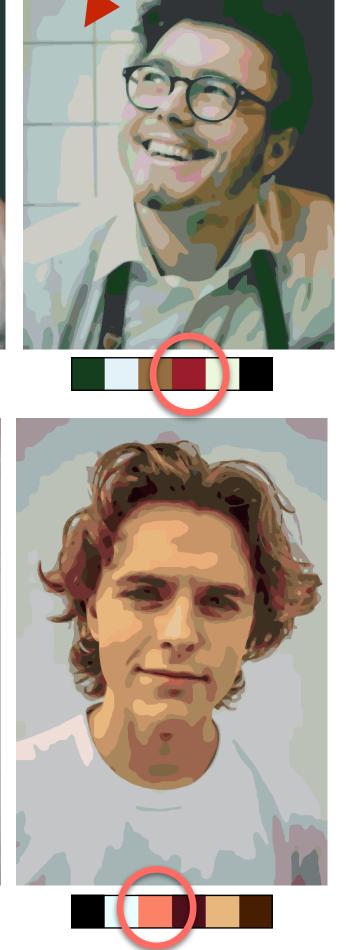




-

Posterized image (K-means clustering to eliminate outliers) Posterized image (without K-means clustering to eliminate outliers)









Input

Direct K-means clustering result







-

Posterized image (K-means clustering to eliminate outliers) Posterized image (without K-means clustering to eliminate outliers)









Input

Direct K-means clustering result







-

Posterized image (K-means clustering to eliminate outliers) Posterized image (without K-means clustering to eliminate outliers)







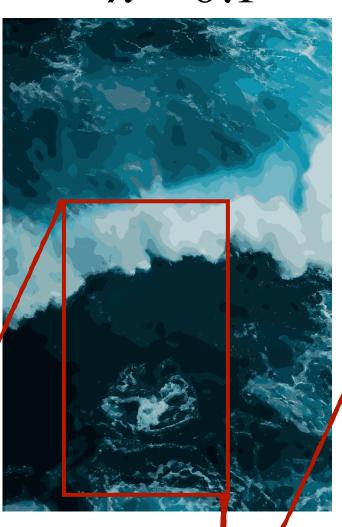


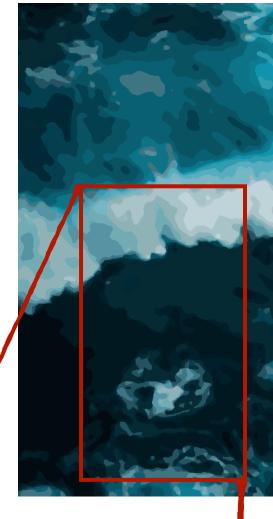
#### Input

 $\lambda = 0.1$ 

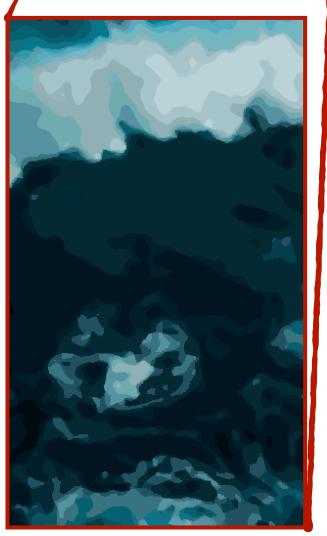
 $\lambda = 1.0$ 



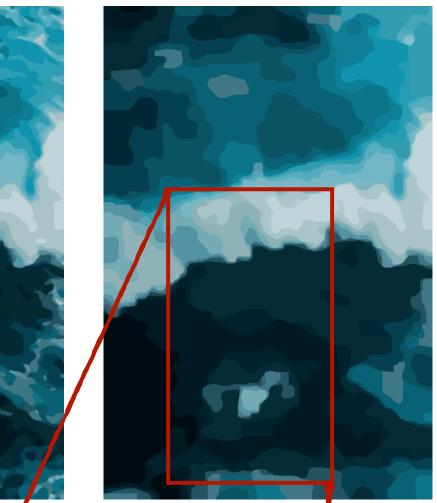


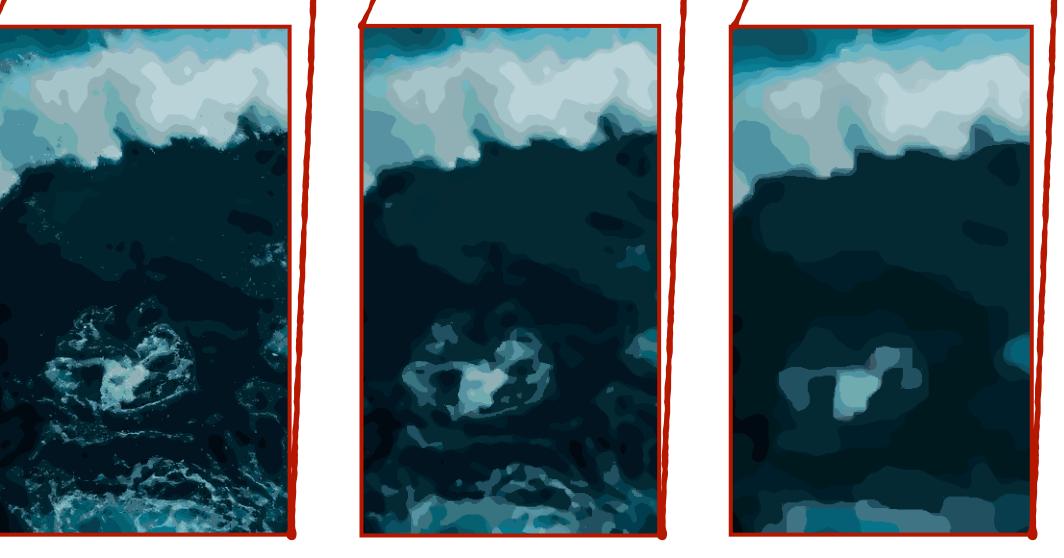






#### $\lambda = 3.0$





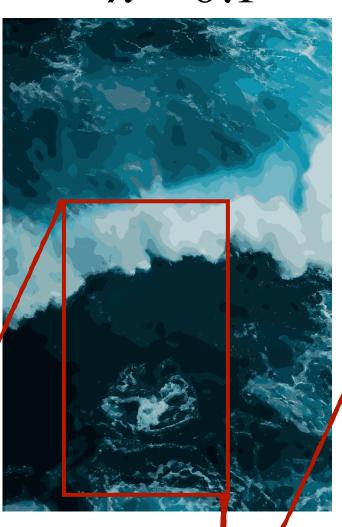


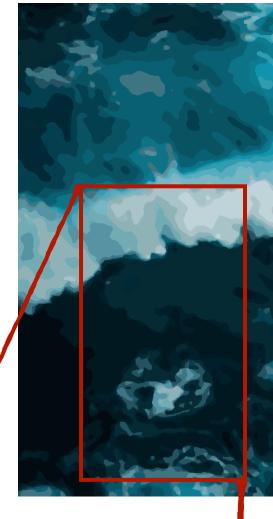
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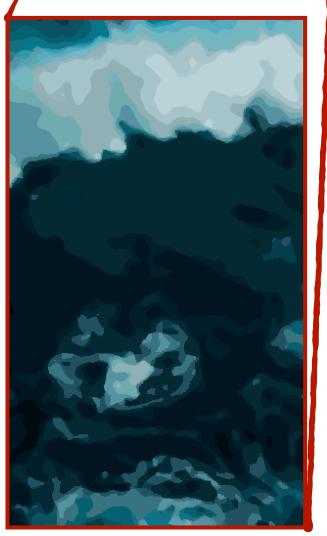
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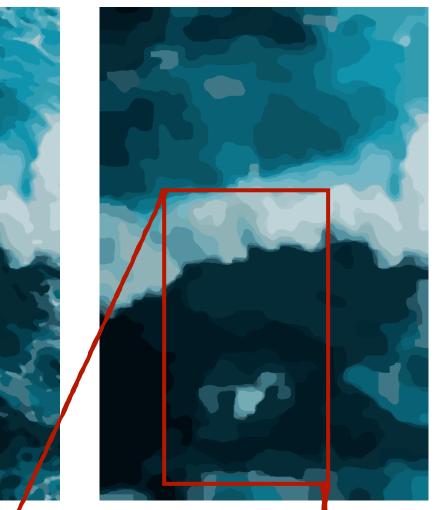


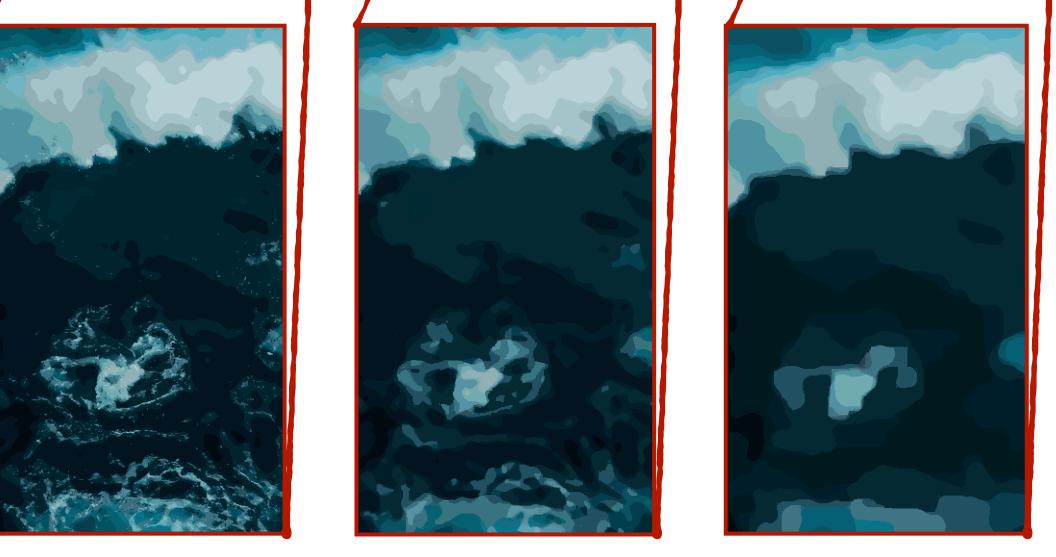




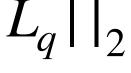


 $\lambda = 3.0$ 





 $\min \sum ||f_p - I_p||_2 + \lambda \sum ||L_p - L_q||_2$  $p \in I$  $p,q\in N$ 



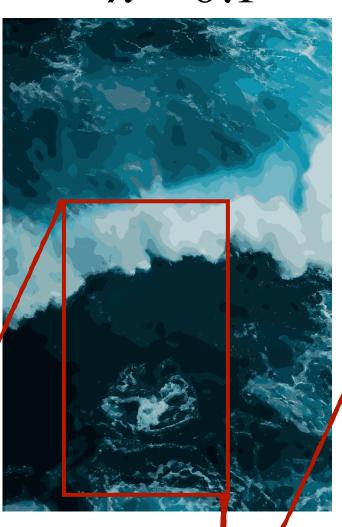


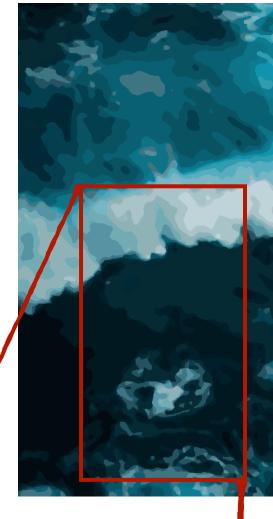
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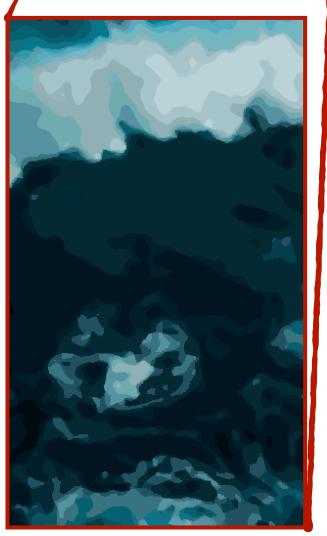
 $\lambda = 1.0$ 



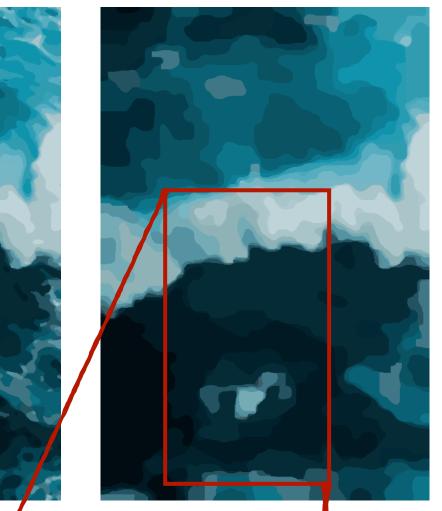


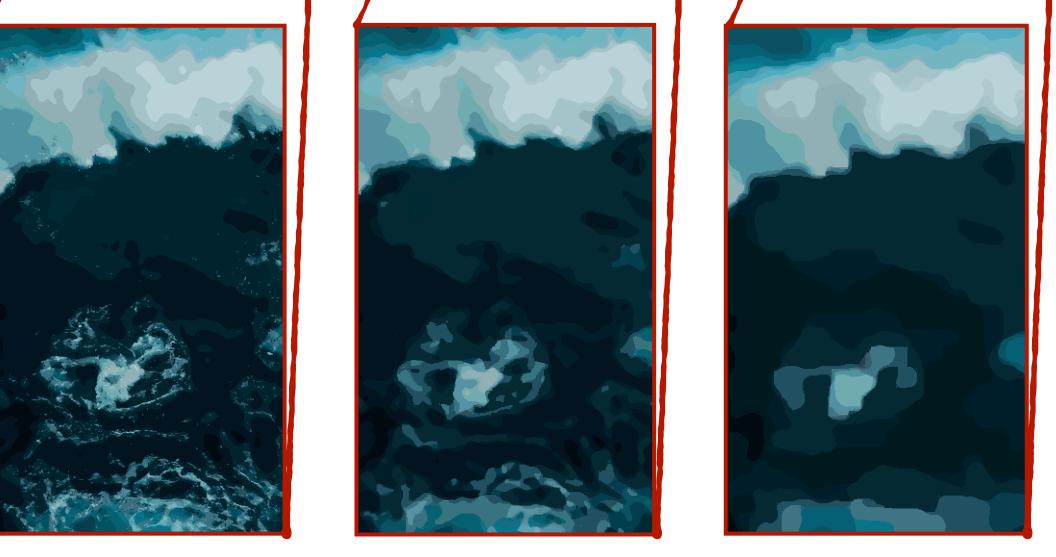






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